A Required Yield Theory of Stock Market Valuation and Treasury Yield Determination

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03. June 2003

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First Version: June 2003
Current Version: June 2009

Published in *Financial Markets, Institutions and Instruments, 18 (1), 2009: 27-88.*

[♣] We thank Ron Balvers and David Smith for helpful comments. All remaining errors are of course our own. This article is a substantive revision of a previous paper circulated under the title “A General Theory of Stock Market Valuation and Return”.
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Abstract

Stock market valuation and Treasury yield determination are consistent with the Fisher effect (1896) as generalized by Darby (1975) and Feldstein (1976). The U.S. stock market (S&P 500) is priced to yield ex-ante a *real after-tax* return directly related to real long-term GDP/capita growth (the *required yield*). Elements of our theory show that: 1) real after-tax Treasury and S&P 500 forward earnings yields are stationary processes around positive means; 2) the stock market is indeed priced as the present value of expected dividends with the proviso that investors are expecting fast mean reversion of the S&P 500 *nominal* growth opportunities to zero. Moreover, 3) the equity premium is mostly due to business cycle risk and is a direct function of below trend expected productivity, where productivity is measured by the growth in book value of S&P 500 equity per-share. Inflation and fear-based risk premia only have a secondary impact on the premium. The premium is always positive or zero with respect to long-term Treasuries. It may be negative for short-term Treasuries when short-term productivity outpaces medium and long run trends. Consequently: 4) Treasury yields are mostly determined in reference to the required yield and the business cycle risk premium; 5) the yield spread is largely explained by the differential of long-term book value per share growth vs. near term growth, with possible yield curve inversions. Finally, 7) the Fed model is partially validated since both the S&P 500 forward earnings yield and the ten-year Treasury yield are determined by a common factor: the required yield.

Keywords: Fisher Effect, Required Yield, Earnings Yield, Equity Premium, S&P 500 Valuation, Fed Model, Treasury Yields, Yield Spread, Productivity, and Book Value of Equity per Share Growth.

JEL: G12

A few key principles have fundamentally shaped the way financial economists think about asset pricing. It is a broad consensus in the profession that stock market prices must be closely related to the present value of expected dividends or free cash flows to equity (Williams, 1938); that the equity premium must be related to the risk differential between Treasuries and stocks (Goetzmann and Ibbotson, 2005); and that ex-ante interest rates must factor a compensation for expected inflation (Fisher, 1896). While these principles are considered cornerstones of Finance, it is quite confounding that the cumulated empirical record of the past four decades has delivered only little support for these principles. Far from elucidating how these principles manifest in the data, the Finance field has faced one puzzle after another regarding the behavior of stock prices and interest rates.

- Puzzle #1: Why are stock market prices more volatile than expected dividends (Shiller, 1981; Campbell and Shiller, 1988a and 1988b)?
- Puzzle #2: Why is the equity premium not accounted for by standard measures of risk and our best asset pricing models (Mehra and Prescott, 1985; Kocherlakota, 1996; Mehra, 2003)?
- Puzzle #3: Why aren't stocks behaving as an inflation hedge instrument, as common sense would dictate (Bodie, 1976; Nelson, 1976; Feldstein, 1980; Fama, 1981; Geske and Roll, 1983)?
- Puzzle #4: Why aren't stock returns (Ritter, 2005) and Treasury yields more directly connected to measures of productivity/economic growth?
- Puzzle #5: Why does the yield spread appear to be a good predictor of real economic cycles (Harvey, 1989; Mishkin, 1991; Estrella and Mishkin, 1996; Kozicki, 1997)?
- Puzzle #6: Why do Treasury yields and the stock market earnings yield appear to behave as non-mean reverting processes (Tatom, 2002; Weigand and Irons, 2007) when stock market returns are found to be mean-reverting in some instances (Balvers, Wu and Gilliland, 2000)?
- Puzzle #7: Why is the so-called Fed model (Lander, Orphanides and Douvogiannis, 1997) linking government bond yields with market P-E ratios found to be a global empirical regularity (Thomas, 2005), in spite of its perceived logical flaws (Asness, 2003)?

Much progress has been made in the areas of stock market valuation (Ohlson, 1995; Dechow, Hutton and Sloan, 1998; Lee, Myers and Swaminathan, 1999; Lee and Swaminathan, 1999; Bakshi and Chen, 2005 and Ohlson and Juettner-Nauroth, 2005) and bond pricing (Vasicek, 1977; Cox, Ingersoll and Ross, 1985; Heath, Morton and Jarrow, 1992 and Ang and Piazzesi, 2003). However, no unified stock and bond market valuation theory has been offered, which resolves the empirical puzzles noted above.

In this article, we introduce a general theory for valuing a broad market index (S&P 500) and for determining the yield on Treasuries of various maturities. Required Yield Theory (RYT) demonstrates that the behavior of stock market prices/returns and Treasury yields is consistent with the cornerstone principles stated above. In other words, we are able to provide an explanation for each of the seven puzzles listed above based on these well-accepted principles.

Required Yield Theory is founded on the Fisher effect (1896) generalized by Darby (1975) and Feldstein (1976). The two latter authors analyze how nominal interest rates are impacted by personal taxes, given that investors want to earn a constant real *after-tax* return ex-ante. We go a step further and argue that this real after-tax return is related to long-term real GDP/capita growth, which we term the *required yield*.

To value the S&P 500, we express the index's price as a function of two components, 1) a perpetuity based on after-tax forward earnings, and 2) the after-tax present value of growth opportunities. Our approach parallels Ohlson and Juettner-Nauroth's (2005) abnormal earnings growth model. The difference is that we account for the effect of personal marginal taxes.

We introduce a novel condition that the present value of expected growth opportunities mean-reverts to zero, which is consistent with the evidence of mean reversion of aggregate earnings growth and corporate profitability (Fama and French, 2000), as well as the evidence in favor of mean reversion of returns (Balvers, Wu and Gilliland, 2000). We derive a compact formula for the present value of growth opportunities and the stock market price as a function of deviations of the growth rate of book value of equity-per-share from long-term GDP/capita growth, and the speed(s) of mean reversion.

We argue that the marginal investor is the highest bidder for the index and thus gets a *minimum* acceptable after-tax real expected return from equities, which turns out to be the required yield. The reason that the required yield is an absolute minimum return is that in the long run, the S&P 500 real capital gains rate converges to real GDP/capita growth and thus constitutes a floor average yearly return, simply by using a long-term buy and hold strategy.[1] Treasuries also provide an alternative minimum (nominal before-tax) return. Thus, investors arbitrage between the two asset classes to get the best after-tax real return available. At each point in time, the highest bidder puts downward pressure on the index's earnings yield to obtain the greater of two expected returns: the required yield vs. the best real after-tax Treasury yield available. Using quarterly data, our stock market valuation model fits the S&P 500 forward

[1] Section 1014 of Internal Revenue Code of 1954 allows for a stepped up basis of unrealized capital gains upon transfer of stock ownership to descendants, essentially eliminating capital gains taxes.

earnings yield with an adjusted R-squared of 88% over Q4 1953- Q3 2006 and 94% over Q4 1978- Q3 2006. We track the S&P 500 forward earnings yield (inverse P-E) with about 19% more accuracy than the Fed model over the whole sample period.

Because our theory predicts that the S&P 500 is priced in relation to a constant real after-tax return, it is fair to wonder how the equity premium fits in our analysis. While there has been recent progress made to explain the factors at the source of the equity premium (Bansal and Yaron, 2004), we adopt a different approach. Because stocks and bonds are taxed at different rates, we postulate that both the after-tax Fisher effect and the Capital Asset Pricing Model (CAPM) in its after-tax version (Brennan, 1970) hold true. Hence, we capture the (after-tax) equity premium as the difference between the nominal required yield and the after-tax yield on a Treasury yield (for a given maturity).

To understand the risk factors that cause the equity premium, we decompose the equity premium into three components: inflation risk, business cycle risk and fear-based risk premia. The first two types of risk are quantifiable economic risks. Fear-based risk is psychological and due to the perception of possible catastrophic events and cannot necessarily be inferred from real economy forecasts. We then create a measure of business cycle risk based on productivity growth slowdowns. We show theoretically and empirically that the equity premium is principally due to business cycle risk. In other words, there cannot be a sustained premium in excess of the required yield over long-term horizons, given that the fear-based risk and inflation risk typically are transient phenomena.

Specifically, we show that the after-tax premium is mostly accounted for by how much short-term productivity growth falls below longer-term productivity trend(s). In our analysis, productivity growth is measured by the growth of book value of equity per share for the S&P 500, which is akin to the sustainable growth rate of corporate earnings. When the economy under-performs, the "short-term" growth rate of book value falls below the trend, and the premium is positive. In that case, the (business cycle) risk faced by an investor is that the index's sustainable rate of earnings growth may *not* revert to the level anticipated over the investor's hedging horizon. On the other hand, zero or even *negative* risk premia may prevail when near-term productivity outpaces the trend(s). Because S&P 500 book value per-share growth closely follows GDP/capita growth, an immediate consequence of this approach is that our measure of the equity risk premium increases during economic contractions and shrinks during economic expansions. This result is consistent with the stylized fact that the equity premium is indeed countercyclical (Ferson and Harvey, 1991).

An implication of this analysis is that it allows for the determination of Treasury yields. This constitutes a validation test for our equity premium approach. Long-term Treasuries return less than the required yield after-tax when the risk premium associated with the business cycle is positive; otherwise, they return exactly the required yield. As long-run productivity growth tracks long-run GDP/capita growth, the after-tax premium for the thirty-year bond must essentially be zero.[2] In our model, short-term Treasuries may return *more* than the required yield in periods of abnormally high short-term productivity growth; otherwise, they return less than the required yield when the risk premium is positive.

This approach is empirically validated, as we find that our model fits the actual behavior of the thirty-year, ten-year and one-year Treasuries, with adjusted R-squares over 66% in all cases. We also find that the yield spread is very well captured by our theory with adjusted R-squares above 58% in the two sample periods: Q4 1953- Q3 2006 and Q4 1978- Q3 2006 . More importantly, our model accurately matches ten yield curve inversions out of twelve over the whole sample period.

A by-product of our theory is that we vindicate the so-called 'Fed' model (Lander, Orphanides and Douvogiannis, 1997) and rationalize why that model works well empirically: both the yields on the stock market and long-term Treasuries are tied to the required yield.

The rest of the article is as follows. In section 1, we offer new evidence that the stock market total return, the forward earnings yield and Treasury yields are stationary processes, on an after-tax and real basis. In section 2, we develop a new valuation model and show that the stock market (S&P 500) can be valued with great accuracy assuming the Fisher effect holds ex-ante and that the after-tax real return is a function of the required yield. In section 3, we show that the required yield is a minimum expected return and ends-up being the yearly stock market return in a steady-state without business cycle risk. This begs the question of how the equity premium fits in our required yield approach. We assert that the equity premium is largely related to the business cycle risk. In section 4, we finalize our stock market valuation formula taking the multiple risk factors into account. In section 5, we focus on characterizing a new measure of the equity premium based on our definition of business cycle risk. In section 6, we draw implications for the determination of the yields on the thirty-, ten- and one-year Treasuries. Section 7 contains a review and discussion of the literature on inflation illusion and the Fed model in light of our

[2] Except when a fear-based premium is present. Glassman and Hassett (1999) argue that the equity premium is essentially zero for long-run investors. While not going as far as we do here, Prescott and McGrattan (2001) argue

results, as well as a discussion of why the ex-post equity premium has been historically large. Section 8 revisits the seven outstanding puzzles of Finance listed previously, in light of our theory. The last section contains our concluding remarks and explores possible extensions.

1. The Fisher/Darby/Feldstein Effect: Evidence for the Mean-Reversion of Stock Market and Treasury Returns on a Real After-Tax Basis

The Fisher effect is a cornerstone of financial economics. While the basic idea can be traced back to several economists a century earlier (Dimand, 1999), Fisher's *Appreciation and Interest* (1896) is the first formal exposition that the *ex-ante* nominal interest rate should compensate investors for anticipated inflation in order to preserve an equilibrium real interest rate. In theory, the equilibrium real interest rate is determined by the marginal productivity of capital and marginal rate of impatience.[3] Darby (1975) and Feldstein (1976) extend this concept and argue that investors seek compensation for the cost of personal taxes in addition to expected inflation in order to achieve a constant ex-ante after-tax real return.[4]

In this section, we shed a new light on the empirical evidence about the Fisher (1896) effect and its generalization by Darby (1975) and Feldstein (1976). We examine the stationarity (or mean-reversion) of time series such as Treasury yields, S&P 500 total return and forward earnings yield (inverse P-E ratio) using after-tax real yields. In other words, we verify whether the stock market and Treasury instruments have a strong statistical tendency to yield a *constant* real after-tax return.

From a broader perspective, the literature has generally been interested in the question of whether stock market returns and interest rates tend to revert back to a mean. Intuitively, if returns are connected to measures of productivity or profitability, which are found in some instances to be mean-reverting, the time series of returns for major asset classes such as stocks and Treasuries should also exhibit mean reverting behavior (Fama and French, 2000; Balvers, Wu and Gilliland, 2000).

Poterba and Summers (1988) find that stock market returns are negatively autocorrelated over the long-term. However, Lo and McKinley (1988) dispute this conclusion and argue that returns follow a random walk using weekly data. Balvers et al. (1997) document that stock market returns do indeed exhibit mean-reverting behavior, by focusing on how several

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that the equity premium measured in reference to long-term debt has been close to zero on a tax-adjusted basis; due to what they view is a small systematic risk premium.

[3] Fisher (1896).

international stock markets return series converge to a common trend. On the other hand, mean-reversion has been empirically rejected for the ten-year Treasury (Bradley and Lumpkin; 1992; Mehra, 1996; Tatom, 2002) and for the stock market earnings yield and its inverse, the stock market P-E (Estrada, 2006; Weigand and Irons, 2007).

The appropriate method to investigate the stationarity of time series is a unit-root test. One widely used procedure is the Augmented Dickey-Fuller test (Hamilton, 1994). This test establishes whether a particular time series behaves as a random walk with no trend, or has a stochastic trend, or mean reverts to a trend or a constant level. For any time series Y_t, the test is implemented by first differencing the series and estimating the coefficients of the following relationship:

$$\Delta Y_t = \beta_0 + \beta_1 T + \delta Y_{t-1} + \sum_{i=1}^{N} \eta_i \Delta Y_{t-i}$$

The drift term is β_0; the slope on the time trend is β_1; the slope on the lagged value is δ and the lagged differenced values coefficients are the η_i s, with N possible lags. The null hypothesis of non-stationarity is that the coefficient δ is zero.

It is worth stressing that for these tests we use data on current marginal (dividend and interest) income and long-term capital gains tax rates. The tax rate applied to the earnings yield is the blended tax rate, where the dividend payout ratio is the weight applied to dividend taxes and the retention ratio is applied to capital gains taxes. While we use the standard Survey of Professional Forecasters for measuring expected inflation, this data is only available since 1970. For our expected inflation data prior to 1970, we rely on a quarterly macro business survey that began in the late 1940s (Thies, 1986). Appendix A gives a detailed description of the data.

In Table 1, we offer three alternate specifications/regressions for the Augmented Dickey-Fuller test. The baseline test in Panel A assumes no drift and no trend. Panel B shows the test with drift. Finally Panel C shows the test with drift and possible trend. We report the coefficients on the slope of the Dickey-Fuller regressions. The null hypothesis for non-stationarity is that the slope coefficient is zero. We report the Akaike information criterion (AIC) and the Schwartz criterion (BIC) as two model selection statistics.

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[4] Darby (1975) uses a static framework, and Feldstein (1976) studies the steady-state in a growing economy.

Our first finding is that total market return (annualized quarterly total return) does *not* appear to be stationary in nominal terms.[5] In Table 1, the slope coefficients become *less* significant as the lag increases from 0 to 4, while the AIC and BIC statistics are decreasing in value, which indicates better models. We also tested alternative Augmented Dickey-Fuller specifications with no-drift or trend for the total S&P 500 return nominal, by varying the number of lags beyond the values reported in the Table. The optimal number of lags to maximize AIC and BIC criteria is found to be 28, which seems excessive given our sample size. For this specification of 28 lags, the t-statistics value for the slope is 1.17, implying that the null hypothesis of non-stationarity cannot be rejected at the 5% level (critical value = 1.95). This result is not very surprising given the mixed evidence in support of the mean reversion of ex-post stock market returns.

On the other hand, when testing for mean reversion of the *real*-after-tax total market return, we find that the null hypothesis of non-stationarity can be rejected. In particular, Panel B shows that the best model (which minimizes the AIC and BIC criteria) is the model with drift and zero lags.[6] The slope and drift coefficients are both significant at the 1% level. This indicates that ex-post market returns do indeed exhibit a mean reversion property, contrary to what is generally found in the literature. This result is our first finding in support the Fisher/Darby/Feldstein hypothesis.

Notwithstanding, the Fisher effect (1896) may be better assessed by testing the stationarity of *ex-ante* measures of returns such as Treasury yields and the S&P 500 *forward* earnings yield. In that respect, Table 1 documents a novel finding regarding the behavior of Treasury yields and of the S&P 500 earnings yield. For the one-year and ten-year Treasuries and the S&P 500 forward earnings yield, while there is some evidence in favor of stationarity in *nominal* terms, the evidence for stationarity is noticeably stronger once the series are adjusted by removing expected inflation and marginal personal taxes. This again confirms the Darby/Feldstein version of the Fisher effect.

First, when examining the specifications for before-tax *nominal* yields in Panels A and C, we find that none of the slopes coefficients are significant at the 5% level for nominal yields, which could lead us to conclude erroneously that these series are indeed non-stationary. But looking at

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[5] We use annualized quarterly returns to avoid the autocorrelation problems that overlapping yearly returns would create with quarterly observations.

[6] We use the actual GDP deflator to calculate real total market returns. Elder and Kennedy (2001) argue that it makes better economic sense to incorporate a drift term but not a trend term in unit-root tests that involve returns, because these time series should neither be growing nor decaying and are likely to be centered around a positive constant mean (drift term).

Panel B which includes a drift term, we can reject the null hypothesis of non-stationarity at the 5% confidence level for all lag specifications.

Our results contrast with the conclusions reached by Weigand and Irons (2007) and Estrada (2006).[7] Weigand and Irons (2007) use a structure with six to twelve lags applied to monthly data. One possible reason for this difference is that these authors do not apply model selection criteria as we do here to determine the best lag structure. Estrada (2006) tests whether the log of the market P-E ratio is stationary in several western economies. This may be problematic as the P-E ratio can exhibit explosive behavior especially when the market earnings yield drops down near a value of zero and thus is bounded below. In other words, the log of P-E may appear to be non-stationary even though the earnings yield is.

On the other hand, turning to *real after-tax* earnings yield and Treasury series in Panels B and C, we observe that all coefficients are significant at least at the 5% level, and above the 1% level for lags equal to 1 or 0. Applying the procedure of minimizing the information criteria for choosing the best specification, we find that Panel B's specification with a drift term and 0 lag is the best model. From cursory examination of Panel C, it is clear that the trend terms are non-significant for all lags and yields. Hence, the after-tax real yields on Treasuries and the S&P 500 are best described as stationary series around constant non-stochastic means. Indeed, the reason why before-tax *nominal* series exhibit weaker stationary behavior may be due to the fact that inflation and tax rates have exhibited clear downward trends over the period. Figure 1 illustrates the behavior of the forward earnings yield and the ten-year Treasury on an after-tax and real basis.

Figure 1 (About here)

Each return series converges to the mean-reversion constant given by the formula Drift/(-Slope). This calculation gives 1.68% for the after-tax real ten-year Treasury, 1.25% for the after-tax real one-year Treasury, 2.21% for the S&P 500 forward earnings yield and 5.15% for the real after-tax market (S&P 500) return. This compares to the actual respective sample mean values of 1.73%, 1.29%, 2.30% and 5.23%.

It is interesting to note that the value of the mean-reversion constant for the after-tax real forward earnings yield is very close to the value of long-run average real GDP/capita growth, which equaled 2.24% over the period 1929-2001 and 2.03% over 1929-2006. While this

[7] Although the sample in the former study ends in 2003 and the second study sample ends in 2005, we do not believe this to be the main reason for the difference in results, since these periods along with our sample period, contain the market correction and ensuing resurgence that took place in the U.S. following the bubble of the late 1990s.

observation may appear coincidental, in the rest of the article, we develop a new theory of stock market valuation and Treasury yield determination to show that there is indeed a connection between Treasury yields, the forward earnings yield and long-term real GDP/capita growth, and that this connection is central for understanding how these assets are valued.[8]

Table 1 (About here)

2. Stock Market Valuation and the Mean Reversion of Growth Opportunities

In this section, we develop a new formula for valuing a stock market index. We begin by expressing the index price as the sum of two components. The first component is a perpetuity based on after-tax one-year forward earnings and the second component is the after-tax present value of expected growth opportunities. This approach is fairly standard. However, the novelty is that we use the property of mean reversion of the present value of growth opportunities to zero at the market level. This property allows us to determine the market index price and forward earnings yield as a function of expected inflation, investors' marginal tax rates and the expected growth rate of book value of equity per share.

2.1. Taxes, growth opportunities and stock market valuation

Our starting point is the conventional formula for the after-tax expected return on a common stock applied to a stock market index:

$$k_{t+1}^{E} = \frac{(1-\tau_{d\ t+1})D_{t+1}^{E}}{P_t} + \frac{(1-\tau_{c\ t+1})(P_{t+1}^{E} - P_t)}{P_t} \tag{1}$$

The variable k_{t+1}^{E} is the after-tax expected nominal return. The variable P_t is the actual stock market price at time t, P_{t+1}^{E} is the expected price and D_{t+1}^{E} is the expected dividend next period. The parameters $\tau_{d\ t+1}$ and $\tau_{c\ t+1}$ respectively are the marginal dividend income and capital gains tax rates.[9]

Equation (1) simply states that the after-tax market return is the sum of the after-tax dividend yield plus the after-tax capital gains. It is worth noting that equation (1) is *fully consistent* with

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[8] On the other hand, the real after-tax total market return is much greater than the value of average real GDP/capita growth. We explain in Section 7 how inflation and tax trends can account for this large ex-post return in excess of real GDP/capita growth.

[9] All expectations are conditional on the information available at time *t*. Note that later in the empirical section, we assume that dividends are taxed at the current income tax rate and that capital gains tax rates are long-term rates. This conforms to gains distribution regulations that apply to the mutual fund industry. However, the model is

the price being determined by the present value of expected dividends. Moreover, this formulation allows for time-varying expected nominal returns. Let $b_{t+1} = \frac{D_{t+1}^E}{e_{t+1}^E}$ represent the expected dividend payout ratio. Rearranging equation (1) to bring the price on the left-hand side, we find that the fair value of the stock market has two components:

$$P_t = \frac{\left(1-\tau_{d\ t+1}b_{t+1} - \tau_{c\ t+1}(1-b_{t+1})\right)e_{t+1}^E}{k_{t+1}^E} + \frac{(1-\tau_{c\ t+1})\left[P_{t+1}^E - P_t - (1-b_{t+1})e_{t+1}^E\right]}{k_{t+1}^E} \tag{2}$$

The first term on the right-hand side of (2) anchors the market price on a perpetuity, which depends on expected forward earnings e_{t+1}^E.[10] The second term on the right-hand side of (2) measures the after-tax expected present value of growth opportunities (PVGO) for the index. The PVGO is the capitalized difference between expected capital gains and retained earnings. Let us denote this second term by $PVGO_{t+1}^E = \frac{\left[P_{t+1}^E - P_t - (1-b_{t+1})e_{t+1}^E\right]}{k_{t+1}^E}$, and let the blended marginal tax rate (or weighted average marginal rate) be denoted by $\tau_{t+1} = \tau_{d\ t+1}b_{t+1} + \tau_{c\ t+1}(1-b_{t+1})$, where the weights are given by the payout ratio b_{t+1} and its complement, the plowback ratio $(1-b_{t+1})$. Thus, equation (2) can be rewritten as:

$$P_t = (1-\tau_{t+1})\frac{e_{t+1}^E}{k_{t+1}^E} + (1-\tau_{c\ t+1})PVGO_{t+1}^E \tag{3}$$

Equation (3) is the time-varying after-tax version of the standard steady-state formula found in most investments textbooks (Reilly and Brown, 2006). In the next subsection, we analyze the mean-reverting behavior of the present value of expected growth opportunities $PVGO_{t+1}^E$ to further pinpoint the market price and the earnings yield, each as a function of economic fundamentals.

2.2. Mean reversion of expected growth opportunities and optimal dividend policy

A well-accepted rule of rational capital budgeting is that corporations should a priori reject (and shut-down) projects having negative net present values. While in theory this selection process

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general enough to account for tax-deferrals if needed; in that case, the correct tax rates to apply are effective yearly tax rates.

[10] Anchoring stock valuation on a forward earnings perpetuity is not new. Ohlson and Juettner-Nauroth's (2005) develop an abnormal earnings growth model to value a market index. Their model is a reformulation of the dividend discount model, which specifies the value of a stock in reference to this perpetuity, and explicitly models the source of growth opportunities.

should result in corporations maintaining positive PVGOs, it is possible that some companies in the early stages of their lifecycle have growth options that are no-longer viable. Some mature companies may suddenly see the demand for their product shrink. In these cases, the firm's PVGO may temporarily end-up being negative, and if prolonged, this situation eventually leads to business closure.

In that respect, S&P 500 index companies possess several interesting features. First, these are mostly mature firms. Given the stage of their lifecycle, mature firms generally have a harder time finding projects with positive net present values due to competitive pressures. Thus, positive abnormal growth is harder to sustain for the index as a whole. Notwithstanding, mature companies exhibiting signs of decay are replaced in the index by new ones having positive growth opportunities. Henceforth, the index continuously showcases a high proportion of thriving mature companies.[11]

On the other hand, as discussed above, the index's PVGO may temporarily become negative due to adverse business cycle conditions. Because mature firms are characterized by a lower systematic risk as compared to younger firms (Grullon, Michaely and Swaminathan, 2002), they are more able to successfully implement cost-cutting measures and maintain adequate profitability in the midst of a down cycle. These companies can be expected to emerge back into positive growth opportunity range during expansionary phases.

Thus, the index's PVGO should on average follow the business cycle and gravitate toward a zero value. The index should generate positive PVGOs during expansions and exhibit shorter spells of negative PVGOs right before, or during recessions. We posit that investors are aware of this tendency and expect the index's PVGO to mean revert to zero over time. Furthermore, the mean reversion should be faster from the negative PVGO range. Figure 2 illustrates that the *actual* S&P 500 PVGOs closely followed the business cycle over the period Q4 1953- Q3 2006.[12]

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[11] The average company stays in the S&P 500 for about 10 years. While this replacement process may lead to some frictions or costs for managing the index because of buying more expensive newer companies and needed adjustments to the divisor, we believe that these adjustments do not disrupt the fundamental property that the S&P 500 growth mimics average economic GDP growth.

[12] The construction of this variable is described in Appendix A. Fama and French (2000) argue that profitability as measured by NOPAT/Assets is mean-reverting with a faster speed from below trend. Balvers, Wu and Gilliland (2000) find that equity returns in eighteen international stock markets are mean reverting within 3 ½ years on average as compared to about 2 ½ years for Fama and French (2000). Ohlson and Juettner-Nauroth's (2005) argument is that abnormal growth occurs when expected earnings per share growth (cum dividend being reinvested) outpaces the required return. In that case, price converges to book value per share and the ROE matches the required return (Reilly and Brown, 2006). Philips (1999) documents that the S&P 500's ROE has been close to the required return on average.

Figure 2 (About here)

A second reason to expect mean reversion of the index's PVGO has to do with the interaction of dividend policy with expected growth opportunities. It is often stated that companies with (positive) growth opportunities should pay little to no dividends, whereas companies experiencing or on the verge of experiencing a negative present value of growth opportunities should increase their dividend payout.

Therefore, an obvious candidate for an optimal corporate dividend policy is to: 1) Pay *all* earnings as dividends (or repurchase shares), when the current PVGO is negative; 2) Retain earnings up to the point where the marginal investment has zero net present value; when the current PVGO is positive. In either case, this policy helps to drive the PVGO back to zero for S&P 500 firms, as the set of positive net present value projects is shrinking for these companies.[13]

We model the *formation* of PVGO expectations as a mean-reverting process to zero:[14]

$$PVGO_{t+1}^{E} = (1-\gamma_i)PVGO_t \text{ with } \gamma_i = \gamma_1 \text{ when } PVGO_t \geq 0\text{; and } \gamma_i = \gamma_2 \text{ when } PVGO_t < 0 \quad (4)$$

Again, the variable $PVGO_{t+1}^{E}$ represents the expected PVGO conditional on period *t* information. The term $PVGO_t$ represents the actual PVGO at time *t*, and the parameters $0 \leq \gamma_1 \leq \gamma_2 \leq 1$ represent the respective annual speeds of mean-reversion to zero, with γ_1 being the reversion speed from above ($PVGO_t > 0$) and γ_2 from below ($PVGO_t < 0$). We assume that these parameters are constant. In Appendix B, we show that from any point in time *t*, when *future* tax

[13] See recommendation #2 of Allen and Michaely (2003). Our hypothesis is fully consistent with the Free Cash Flow hypothesis of Jensen (1986) and with the Maturity Hypothesis of dividend policy (Grullon, Michaely and Swaminathan, 2002). The papers by Gordon (1963) and Walters (1963) are the standard references for the thesis that dividend policy should be affected by the presence of varying growth opportunities. On the other hand, Modigliani Miller (1961) irrelevance of dividends proposition assumes that investment policy is set in advance and independent of dividend policy. For a comparison of the two competing theses, see Brennan (1971). In this paper, we implicitly assume that as the rate of investment increases and firms retain a larger portion of their earnings, decreasing returns set in and profit rates decline for *mature* firms. Obviously our optimal dividend scheme is subject to possible counter-incentives due to the tax code. For example, the retention ratio may be higher than optimal when dividends are taxed at a much higher rate than capital gains. Our argument also relies on the fact that most S&P 500 companies have a stable capital structure. Thus, we sidestep the fact that they could be paying higher dividends and still finance the same growth using cheaper debt.

[14] Contrary to what may appear at first glance, this set-up does not lead to a circularity problem. Our final valuation formula does *not* require us to know what the actual $PVGO_t$ is and thus what the current price is, which we are trying to predict. In practice, we use *last quarter's* PVGO instead of the current PVGO to signal whether the current PVGO is positive or negative, since once again to find the current PVGO, we would need to know the current price.

rates and payout ratios are expected to remain constant, the combination of equations (3) and (4) leads to the following expression for the expected PVGO: [15]

$$PVGO_{t+1}^{E} = (1-\tau_{t+1})\frac{e_{t+1}^{E}}{k_{t+1}^{E}} \times \frac{(1-b_{t+1})\left[\dfrac{g_{t+1}^{E}}{(1-b_{t+1})} - \dfrac{k_{t+1}^{E}}{(1-\tau_{t+1})}\right]}{k_{t+1}^{E} + (1-\tau_{c\ t+1})\gamma_i} \quad \text{for } i = 1,2 \tag{5}$$

Equation (5) states that the expected PVGO equals the after-tax forward earnings perpetuity times an adjustment factor that accounts for the deviation of the before-tax required return $k_{t+1}^{E}/(1-\tau_{t+1})$ away from the expected sustainable ROE (equal to the expected growth rate g_{t+1}^{E} of book value divided by the retention ratio).[16] Note that the greater the ROE as compared to the required return in equation (5), the greater the expected PVGO, which is a standard result. A high reversion speed γ_1 (γ_2) reduces the size of positive (negative) growth opportunities.

To simplify notations, it is useful to refer to the adjustment factor in the PVGO formula (5) as the expected *abnormal earnings growth* component denoted by $AEG_{i,t+1}^{E}$, and given by the relation $AEG_{i,t+1}^{E} = \dfrac{\left[g_{t+1}^{E} - \dfrac{(1-b_{t+1})k_{t+1}^{E}}{(1-\tau_{t+1})}\right]}{k_{t+1}^{E} + (1-\tau_{c\ t+1})\gamma_i}$ for $i = 1,2$. Thus, equation (5) becomes:

$$PVGO_{t+1}^{E} = (1-\tau_{t+1})\frac{e_{t+1}^{E}}{k_{t+1}^{E}} \times AEG_{i,t+1}^{E} \quad \text{for } i = 1,2 \tag{6}$$

In other words, the PVGO equals the present value of capitalized abnormal earnings growth as shown for example in Ohlson and Juettner-Nauroth (2005). Here, we have introduced a novel condition that growth opportunities mean-revert to zero. One advantage of modeling growth opportunities that way is that one avoids the arbitrary selection of the number and length of intermediate growth stages leading to the terminal steady-state growth price (Lee, Myers and

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[15] Based on actual PVGO (see Appendix A for construction of this variable), we estimate the mean reversion parameters values at γ_1 = 43.6%, and γ_2 = 68% over the whole sample period. These parameters are significant at the 1% level. Our positive mean reversion parameter γ_1 is close to the estimate by Fama and French (2000) who find that the average mean reversion of profit rates is about 38% per year. They also find that the reversion speed from below is faster than from above. We acknowledge that assuming constant future payout ratios conditional on information obtained at time *t* is a bit restrictive, as we consider only the case of a one-time adjustment to the payout ratio. However, this adjustment is still in accordance with the optimal dividend policy discussed above.

[16] This difference must be positive (negative) to be consistent with having $PVGO_{t+1}^{E}$ positive (negative). We discuss this constraint in Appendix B.

Swaminathan, 1999; Lee and Swaminathan, 1999)[17]. A similar approach to ours is Dechow, Hutton and Sloan (1998), who use the residual income model of Ohlson (1995) and assume that the ROE of firms fades to the cost of equity.

We determine the stock price by combining equations (3), (4) and (6), and obtain:

$$P_t = (1-\tau_{t+1})\frac{e_{t+1}^E}{k_{t+1}^E} \times \left[1 + (1-\tau_{c\ t+1})AEG_{i,t+1}^E\right] \text{ for } i = 1,2 \tag{7}$$

With $AEG_{1,t+1}^E$>0 and $AEG_{2,t+1}^E$<0. Equation (7) states that the stock price equals the after-personal tax forward earnings capitalized at a rate equal to the required return, times one plus the after-tax abnormal earnings growth factor. It is important to note that equation (7) is only operational as a valuation formula as long as the required return is known a-priori. In the next section, we introduce a set of logical propositions linking investors' bidding behavior to the required return. These propositions lay the foundation for showing that the long-run GDP/capita growth rate (i.e. the required yield) is a key determinant of the required return k_{t+1}^E.

3. The Fisher Effect, Personal taxes, After-Tax CAPM and the Required Yield

3.1. The required yield is a minimum after-tax real expected equity return

In this section, we demonstrate that the required after-tax real stock market return is related to real long-term GDP/capita growth (i.e. the required yield).

> <u>*Proposition 1 (Fisher/Darby/Feldstein Effect on Stock Returns)*</u>*: At each point in time, the stock market index's price is set by the highest bidder (i.e. the marginal investor). The marginal investor will bid the highest price and thus obtain (ex-ante) the minimum acceptable holding period (after-tax real) return. The lower the investor's marginal tax rate, the higher the bid.*

Proposition 1 essentially states that investors behave in accordance with the generalized Fisher (1896) effect (Darby, 1975; Feldstein, 1976) and price stocks to obtain a target minimum real return after-tax. While we postulate that there is such a minimum return, which is common to every equity investor, actual bidding behavior depends on the marginal investor's tax rates. For example, it is possible that the maximum potential bid might be attributable to a long-term investor (e.g. member of a pension fund) who benefits from deferred taxes and therefore willing to pay a higher price than other investors. On the other hand, mutual funds are known to have much higher turnover ratios than pension funds (Hotchkiss and Strickland, 2003). Thus, when

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[17] In these two papers, the authors use forecast earnings for the first three years. Beyond year 3, forecast ROEs are calculated by assuming a gradual fading to median industry value. A terminal value is then generated over a finite horizon, which extends up to eighteen years.

the marginal trader is a portfolio manager trading on behalf of its clients, the members of the fund face a greater tax burden, and thus the actual highest bid may fall short of the potential maximum bid.[18]

> *Proposition 2 (Required Yield): For a long-term investor, the average yearly real capital gains rate obtained by investing in the S&P 500 is equal to long-term real GDP/capita growth (required yield). The required yield constitutes a minimum average after-tax real return, obtainable with zero asymptotic yearly risk.*

The proof has several non-trivial steps. First, in an economy steady-state, it must be true that aggregate corporate earnings grow at the same rate as GDP.[19] Because the S&P 500 market capitalization constitutes a stable fraction of the overall market and the growth of these companies tracks GDP, this argument must hold for the index as well. Secondly, in Appendix C we show that net new equity shares for the S&P 500 grow theoretically and empirically at the rate of population growth in a steady-state. Hence, earnings per share must grow at the rate of GDP/capita. Because the market P-E ratio is constant as well in the steady-state, capital gains must grow at the rate of GDP/capita as well. Finally, a buy and hold strategy with an *infinite* horizon guarantees an average yearly real after-tax return *at least* equal to real long-term GDP/capita growth, given that investors also receive dividends.[20]

However, it is important to note that the above argument while describing the average return does not pinpoint the return obtained by the marginal investor in each period. The next Corollary helps in that matter.

> *Corollary 2.1(The Fisher Golden Path): Assume that the rate of inflation is constant. In the absence of business cycle fluctuations and other (transient) risks, the required yield*

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[18] Tax-efficient mutual funds may reduce their members' tax burden for example by limiting portfolio turnover in order to avoid short-term capital gains taxes. It is also true that heterogeneous tax brackets may influence the argument. However, we assume that average marginal tax rates are the relevant rates, as participation in the stock market has spread to the general public with the enactment of the Investment Company Act in 1940.

[19] The argument is based on Kaldor's (1961) stylized facts documenting that the U.S. economy has been characterized by a constant nominal (and real) GDP growth rate that equals the growth rate of capital, with a stable factor income distribution (labor vs. capital). Diermeier, Ibbotson and Siegel (1984) use a similar argument that financial wealth grows at the same rate as GDP to conclude that the supply of aggregate returns on capital is set by the productivity of business. In the long-run, the growth of aggregate equity capital cannot be disconnected from the rate of GDP, because the corporate sector is the source of GDP growth.

[20] In the long-run, it is a known result that the standard deviation of annualized continuously compounded returns decreases in proportion to the square root of the time horizon, thus risk as measured by the *yearly* standard deviation of stock market returns over the horizon decreases to zero in the limit. That is, on average the rate of real GDP/capita is achievable as a minimum return without risk. Section 1014 of Internal Revenue Code of 1954 allows assets to be transferred from one generation to the next on a stepped-up basis, which effectively eliminates capital gains taxes on unsold assets. In practice, achieving this return as an *actual* minimum return will be made easier when the index is bought at P-E ratios less or equal to the long-run average level. Other strategies such as spreading initial investment over time will produce a portfolio's average P-E ratio close to the market average P-E. In addition, a progressive divestment strategy will reduce the risk to zero faster over the long run.

constitutes an absolute minimum after-tax annual real return achievable by a marginal (short-term or long-term) investor in the S&P 500 year to year.

This Corollary simply states that in the absence of business fluctuations/risk, the economy simply hums along its steady-state growth path, and in that respect, the *real* capital gains rate matches real GDP/capita growth year to year. In Appendix D, we argue that dividends are always large enough to pay for taxes on capital gains (per share), so that the required yield does indeed constitute a minimum after-tax yearly real return obtainable by any investor, not just investors following the buy and hold strategy described above.

Thus, in this ideal state we call the Fisher Golden Path any marginal investor/highest bidder of the market index with either a short-term or long-term horizon must *exactly* earn real GDP/capita growth on an annual basis. Furthermore, because risk is absent, this return must be the riskless rate on Treasuries as well. Let us now turn to the question of how risk is incorporated in a more realistic economy subject to the business cycle.

3.2. Business cycle risk, the CAPM and the required yield

In the previous section, while we have shown that the required yield is a minimum long-run after-tax real return in the absence of business cycle fluctuations, we have not yet answered the critical question of how risk affects the required return. To help answer this question, we first acknowledge the key role that business cycle fluctuations play in generating corporate earnings risk. First, we propose a new definition of business cycle risk:

<u>Definition (Business Cycle Risk)</u>: is the risk that while the economy is entering a downward phase of the business cycle or during a business downturn, productivity growth will <u>not</u> rise to the projected level consistent with mean reversion to the trend, and over the investor's relevant horizon.

Burmeister, Roll and Ross (1994) define risk as unanticipated changes in the level of the overall business activity. Here, we posit that rational economic agents do anticipate the mean reversion of macroeconomic productivity.[21] In this article, we define risk as the possibility that upward mean-reversion does not occur over the horizon relevant to the investor. The relevant horizon is defined as the one over which the investor wants to hedge against risk.

Obviously, this risk would be eliminated if the business cycle did not exist. Then, the required yield would be the benchmark return that investors are targeting. The reason we are focusing exclusively on business cycle risk is that business cycle fluctuations are the chief cause

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[21] Given that the business cycle is caused by a series of exogenous shocks to the economy, and that these shocks are unpredictable, forecasters should find it virtually impossible to predict troughs and peaks of the business cycle.

of systematic risk for corporate earnings growth and thus must logically be the source of the equity risk premium, almost by definition.[22] The next step is to analyze how the required stock market return is determined in relation to the risk premium on an after-tax basis.

To proceed with the analysis we must first point out that the Finance field appears to hold two separate views of how the required return on stocks is determined. In one strand of the literature, ex-ante returns are assumed to conform to the Fisher effect, which has often been interpreted to say that the real ex-ante return is a *constant over time* (Fama and Schwert, 1977; Boudoukh and Richardson, 1993). On the other hand, the literature based on the Capital Asset Pricing Model (CAPM) treats the equity premium as a building block in the determination of the required stock return, which is viewed as time-varying (Ibbotson and Chen, 2003; Fama and French, 2002). Our theory offers a synthesis of these two distinct but not mutually exclusive views. To integrate the equity premium in our analysis we postulate that the Fisher/Darby/Feldstein effect holds in conjunction with an after-tax version of CAPM similar to Brennan's (1970).[23]

> *Proposition 3 (After-tax Fisher Effect and After-tax CAPM Combined): Assume that there are no other risks than business cycle risk. Year to year on an after-tax basis, the marginal investor expects to earn the nominal required yield (long-run real GDP/capita growth + one-year inflation forecast), which compensates him for taking on business cycle risk, if and only if the Nominal Required Yield = Nominal Risk-Free Rate + Business Cycle Risk Premium.*

Proposition 3 essentially states that the required yield is the annual real return required by investors if and only if the business cycle risk premium (equity premium) coincides with the difference between the nominal required yield and the nominal risk-free rate on an after-tax basis.[24] This proposition is logically trivial given our premise. However, it is not economically obvious because we assert that the CAPM return is constant in real after-tax terms. In the standard CAPM approach the market return varies on a real-after tax basis with the magnitude of the equity premium. Here we postulate that the required yield already includes the business cycle premium.

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[22] Longstaff and Piazzessi (2004) argue that corporate earnings are an important factor in explaining the equity premium due their volatility in response to business cycle shocks.

[23] Boudoukh, Richardson and Withelaw (1994) show that the standard dynamic asset pricing model of Lucas (1978) is compatible with the Fisher effect. However, they do not address the issue of the equity premium and the effect of taxes and do not tie the real return to long-run GDP/capita growth. While Brennan (1970) works with the effect of personal tax rates on the original CAPM framework, he expresses his results on a *before-tax* basis.

[24] We choose to present the proposition in terms of *nominal* returns because the relevant inflation rate may *vary* depending on the marginal investor's horizon while the nominal required yield (on the equity index) is defined using *one-year* inflation forecast.

Proposition 3 sets-up a joint hypothesis about the stock market ex-ante return being the required yield and the equity premium being exclusively caused by business cycle risk. We postulate that investors regard the Fisher Golden Path as the ideal state to come back to. While there is risk associated with delayed mean reversion, rational market participants still conjecture that mean reversion will take place and therefore that over the long-run, business cycle risk should not impede the returns promised along the Fisher Golden Path.

The fact that business cycle risk jointly determines the risk-free rate in relation to the required yield is analogous to an insurance scenario where investors must sacrifice some upside return in exchange for the certainty of not losing real principal. This insurance is achieved by investing in the Treasury instrument that provides hedging over the pertinent horizon. It remains to show that Proposition 3 is empirically validated. This issue will be addressed in Section 5. In this next subsection, we pursue our analysis of the other risk factors influencing the equity risk premium.

3.3. Treasuries arbitrage, and other risk factors

It is important to emphasize at this point that investors do have access to an alternate minimum nominal return, by investing in Treasury instruments.[25] Thus, it is natural for investors to arbitrage the stock market index vs. Treasuries by comparing the required yield with the minimum after-tax real yields expected from Treasuries, and demand to earn the greatest available return. Proposition 3 above asserts that the equity risk premium is associated with Treasury yields that are lower than the required yield. However, we recognize that on an after-tax and real basis, Treasuries may in some instances return more than the required yield, for example when an inflation risk premium is present. Other factors such as financial assets tax reforms or yield curve inversions may also drive the level of Treasury yields above the required yield.

In other words, other types of risks and factors may be present besides business cycle risk, which in turn affect the equity premium. These risks may drive investors to bid a lower market price than the price justified by the required yield. At the macro level, these types of risks fall into two remaining categories: 1) inflation risk and 2) confidence risk or what we call fear-based

[25] We assume here that these Treasury yields with different maturities are directly substitutable from the marginal investor’s standpoint given his /her hedging horizon. We realize this is a rough approximation because of price and reinvestment risk.

risk.[26] Inflation risk is due to the uncertainty associated with future unanticipated inflation shocks. As shown previously, in the long-run, the growth of earnings per share is tied to GDP/capita growth, i.e. the required yield. Similarly, in the long run, the nominal growth of corporate earnings is essentially indexed with actual inflation, as costs and revenues must on the whole rise with inflation.[27] Although investors know that real long-term GDP/capita growth is independent of inflation shocks, they will still include an inflation risk premium because investors demand a nominal required yield, and unanticipated inflation may affect their ex-post real return. An inflation risk premium is also required on the bond side because the fixed cash flows and terminal value are not protected against unanticipated inflation.[28] On the other hand, while it may be hard to predict the conditions leading to the onset of a fear-based risk premium, the presence of such a premium manifests as a flight to safety and thus drives the yields of intermediate and long-term Treasuries *down* and the required return on stocks *up*.[29]

Let us define the variable $R_{t+1}^{E} = g + \pi_{t+1}^{E} + \Phi_{t+1}$ as the nominal required yield plus the fear premium, where the nominal required yield is the sum of g (long-term real GDP/capita growth rate) plus π_{t+1}^{E} the expected inflation rate over the next year. Let the variable X_{t+1} represent the combination of an inflation risk premium and/or boosts to the minimum real return on the Treasury side for example due to a yield curve inversion. We can then express the after-tax stock market required return as $k_{t+1}^{E} = R_{t+1}^{E} + X_{t+1}$, which comprises the nominal required yield adjusted for the possible presence of a fear-based premium Φ_{t+1} >0 and other premia aggregated in X_{t+1}.

We determine the components of the variable X_{t+1} using the fact that the highest bidder seeks to arbitrage between the minimum return obtainable on the market index vs. Treasuries. Let us denote by r_{t+1}^{j} the *after-tax* nominal expected yield on a Treasury bond with maturity j = *1, 10*. As each Treasury yield already includes an inflation risk premium, the arbitrage condition

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[26] Burmeister, Roll and Ross (1994) define five types of risk. Here, we use three types of risks, business cycle risk, inflation risk and fear-based risk. Our business cycle risk is a combination of Burmeister, Roll and Ross' own definition plus their time horizon risk and market timing risk. Our fear-based risk is akin to their confidence risk. Because we are dealing with widely traded assets (Treasuries and equity indexes) we attribute any potential liquidity premium for the whole term structure to the fear-based premium.

[27] Of course, there are recessionary periods where nominal earnings fall in spite of the presence of inflation pressures (stagflation). An inflation risk premium should still be present then.

[28] Investors can obviously get close to a minimum before-tax "real" return by investing in TIPS. The inflation risk premium has been shown to have a non-trivial effect on U.S. Treasury bonds (Shen, 1998). Conventional wisdom points to the inflation risk premium as having a larger effect on long-term maturity Treasuries.

[29] The net effect of the fear premium on short-term Treasuries should be nil. In the event of a disaster, short-term instruments should a-priori contain a fear-based risk premium. This would lower their yield. However, flight to safety to longer-term maturities then raises their yield back to the status quo.

implies that any excess real return provided by the other kinds of risk premia and factors listed above can be measured by the extra yield that Treasuries provide in excess of the required yield. Hence, we can therefore formulate $X_{t+1} = Max\{r_{t+1}^{1} - R_{t+1}^{E}, r_{t+1}^{10} - R_{t+1}^{E}, 0\}$.[30] At the long-end of Treasury maturities the excess premium over the real required yield is mostly caused by an inflation risk premium. At the short-end of the yield curve, it is caused by yield curve inversions. Thus, the required after-tax stock nominal market *return* must be given by $k_{t+1}^{E} = Max\{R_{t+1}^{E}, r_{t+1}^{1}, r_{t+1}^{10}\}$. In other words, the required *return* on the equity side includes an inflation risk premium and allows investors to earn a minimum return that could temporarily outpace the required yield. This next proposition sums-up the arguments above.

> *<u>Proposition 4 (Non business-cycle risk factors)</u>: The marginal investor seeks a higher after-tax nominal return than the required yield only when the equity premium is due to additional factors/risk(s) other than the risk associated with the business cycle. Such cases are: 1) when there is a fear-based premium or 2) when after-tax nominal Treasuries exceed the nominal required yield due to an inflation risk premium, a major tax reform, or a yield curve inversion.*

4. A New Stock Market Valuation Formula and the Effect of Inflation on Stock Prices

Going back to our central equation (7), substituting the required return and rearranging terms, we can express the market earnings yield (inverse P-E) as follows: [31]

$$\frac{e_{t+1}^{E}}{P_t} = \frac{Max\{R_{t+1}^{E}, r_{t+1}^{1}, r_{t+1}^{10}\}}{(1-\tau_{t+1})\left[1+(1-\tau_{c\ t+1})AEG_{i,t+1}^{E}\right]} \text{ for } i = 1,2 \qquad (8)$$

The marginal investor bids the market price up so that the market forward earnings yield equals the required yield, adjusted for growth opportunities and possible Treasury arbitrages. The bid also incorporates the fear-based premium, under the appropriate conditions.[32]

Our market valuation formula (8) addresses one of the most enduring puzzles of Finance. It shows that a sudden rise in inflation expectations *negatively* impacts the stock market P-E ratio as documented for example by Sharpe (1999, 2001). This empirical finding has been interpreted

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[30] By construction of the variable k_{t+1}^{E}, is a one-year holding period expected return. We include the 10-year Treasury in the analysis, because as we will see later it is consistent with a measure of expected one-year yield, which accounts for mean reversion of productivity.

[31] To be consistent with steady-state valuation our model implies that the dividend payout ratio must equal the blended tax rate in the steady-state. This is intuitive in the sense that the dividend payout is necessary to provide investors with a before-tax return large enough so that the required yield may be obtained after-tax. In a world with zero personal taxes on equity returns, we predict that the payout ratio would *effectively* be zero. Investors would bid stock prices up to eliminate the surplus return that free cash flows provide, this would lead to the same return as if firms were buying back shares. That return would again be the required yield.

in the literature as a failure of stocks to provide an inflation hedge. While we agree with the literature that the stock price fails to be indexed with short-run changes in inflation expectations, it is still the case that the market required return does co-vary positively with expected inflation.

Equation (8) implies that a shift in expected inflation affects market prices because: 1) the Fisher/Darby/Feldstein effect induces investors to want to earn ex-ante the required yield after inflation and taxes; and 2) real growth opportunities are impacted, because the corporate ROE does not adjust upward as quickly as the required nominal return in the event of a jump in expected inflation, which then drives the stock market price down.

The latter effect is important enough to illustrate via a simple example. Assume the book value of equity is \$1,000 and expected earnings are \$20, with zero tax, zero payout and zero inflation, so that the ROE = 2%. Let the required yield match the ROE at 2%. When the rate of expected inflation suddenly rises, for example by 1%, the nominal required yield adjusts upward by the same 1% to a new value of 3%, because of the Fisher effect. However, the nominal ROE of 2% will not rise to 3% instantaneously, since earnings have increased only to \$20.2, whereas to guarantee a ROE of 3%, earnings should have risen to \$30. This means that, independently of real business cycle effects, a spike in expected inflation lowers the PVGO, since the *real* ROE immediately drops in value, then rises up slowly to its new level. In other words, higher expected inflation has the effect of reducing the set of acceptable projects with positive NPVs.

In his book *Stocks for the Long Run* (2002), Jeremy Siegel briefly discusses linking the market P-E ratio to the Fisher/Darby/Feldstein effect. He states on pp. 119-120: "If we assume that investors bid stock prices up or down in response to changing taxes and inflation to obtain the same *after-tax* real return, we can calculate how shifts in these variables affect the P-E ratio."[33] Required Yield Theory makes it clear that this argument is indeed at the heart of understanding stock market valuation.

Our theory allows for short-term or medium-term deviations from the pricing formula (8). These deviations may occur for several reasons: 1) economic, productivity or policy shocks that may lead to revisions of the equity risk premium, earnings forecasts, productivity trends, or inflation expectations not yet reflected in publicly available consensus experts' opinions; 2) short-term noise trading.

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[32] The fear premium is determined in Section 6.

4.1. *Empirical fit of the S&P 500 earnings yield using Required Yield Theory vs. the Fed model*

In this section, we test the valuation formula (8). We empirically analyze the performance of our model on a quarterly basis over the period Q4 1953-Q3 2006 and study how the model stacks-up against the Fed model (Landers et. al, 1997). The data and variables are described in Appendix A. The reason we focus on quarterly data is because it is the highest frequency data available for standard surveys about U.S. inflation expectations over the period studied.

Estimates of real long-term GDP/capita growth (required yield) are reported in Table 2. We report these values over three different horizons and the values range from 1.94% to 2.25%. For our empirical tests we use a value for the required yield of 2.21%, which falls within the narrow range of estimates for long-term real GDP/capita growth and matches the mean reversion constant for the after-tax real earnings yield.

Table 2 (About here)

Although the Fed model has received severe criticism from the academic community (Asness, 2003) is still one of the most widely used model by practitioners to value the S&P 500. A version of the Fed model popularized by Yardeni (2002) states that:

$$\frac{e_{t+1}^{E}}{P_t} = \text{10-Year Treasury Yield} \tag{9}$$

We divide our sample into two periods: Q4 1953- Q3 2006 and Q4 1978- Q3 2006. We choose the second period beginning in 1979 because forward earnings forecasts became available only then. One issue we face is that our nominal variables are not stationary over the second subsample Q4 1978- Q3 2006. This may invalidate our regression analysis because we would not be able to rule out spurious relationships. In Appendix E, we present the regression analysis based on a uniform transformation of our variables. Essentially, we use after-tax real variables.[34] The coefficients of each regression are estimated using the Newey-West estimation procedure. We correct for autocorrelation up to 4 lags, and also correct for heteroskedasticity. The adjusted R-squares and information criteria, our measures of overall fit, are obtained from simple OLS regressions using these transformed variables.

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[33] Siegel also cites McGrattan and Prescott (2000). However, neither Siegel (2002) nor McGrattan and Prescott (2000) develop a full-blown theory based on a comprehensive approach covering the equity premium and the Fisher effect as we do here.

[34] Our tests reject non-stationarity of these series at the 1% level for most variables and 5% level in a couple of instances, for both sample periods. These results are available from the authors upon request.

Table 3 shows the statistical results of the performance of our model vs. the Fed model. It shows that based on the adjusted R-squared criterion the overall best model specification incorporates Treasuries arbitrage and mean reversion of PVGOs for both sample periods. Our model explains 88% of the variability of the market forward earnings yield for the period Q4 1953- Q3 2006 and 94% of the variability over Q4 1978- Q3 2006. One alternate specification assuming that mean reversion is instantaneous ($\gamma = 1$) seems to be doing better for the Q4 1953- Q3 2006 in terms of its predictive ability based on the lowest value for information criteria (AIC and BIC). Other alternatives assuming that Treasuries arbitrage is absent are worse models in terms of fit. Our valuation formula appears to be between 8% and 19% more accurate than the Fed model across these samples based on adjusted R-squares.

It is interesting to note that Required Yield Theory appears to value the S&P500 much more accurately than the Fed model prior to the 1970s. It is well documented that the ten-year T-Note did not track the S&P 500 earnings yield prior to 1970 but moved closely with the earnings yield since then. The slope coefficient on the real after-tax return is close to unity and the slope coefficients on the expected inflation variable are non-significant for the best model specification, as the theory predicts. Figure 3 graphically shows on a *before-tax nominal* basis the difference between the Fed model and our best performing RYT model that incorporates Treasuries arbitrage and mean reversion of growth opportunities.[35]

Table 3 (About here)

Figure 3 (About here)

5. A New Characterization of the Equity Risk Premium on an After-Tax Basis

5.1. The equity premium and the business cycle

In Proposition 3 we defined the after-tax ex-ante equity premium as the difference between the nominal required yield vs. an after-tax Treasury yield (of a given maturity). We now provide an alternate measure of the equity premium based on business cycle risk and show that the required yield approach is consistent with the after-tax CAPM. We construct a new risk measure based on the gap between productivity vs. long-term trend when productivity dips below trend, in accordance with our definition of business cycle risk developed in Section 3. We first show that this measure logically accounts for the equity premium. Such measure would be of little use if it

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[35] Figure 3 shows a discrepancy of our model with actual S&P 500 forward earnings yield during the 1998-2000 market bubble. This may appear inconsistent with our hypothesis of investors wanting a minimum absolute real

did not lead to good empirical predictions. In Section 6, we show that this new measure is a major determinant of actual Treasury yields.

Even though investors bid to obtain the required yield ex-ante, there still is risk associated with short-term stock market volatility, which some long-term investors may want to hedge against, using Treasuries. In that context, risk-compensation has two interpretations: 1) in order to accept risk over the relevant horizon, investors want to earn a premium in excess of the riskless rate; or 2) in order to hedge against risk over the relevant horizon, investors have to pay an insurance premium, which brings their expected return down to the riskless rate.[36] This latter interpretation is consistent with our definition of business cycle risk introduced in Section 3. The risk faced by investors is that below trend corporate productivity does *not* mean revert to its long-run trend with the reversion speed that is anticipated over the hedging horizon.[37] Here, risk is viewed as the opportunity loss of not catching-up to the Fisher Golden Path.

Because the equity premium is usually defined with respect to a specific Treasury instrument, we treat each Treasury's maturity as a preferred horizon over which an investors may hedge and obtain a riskless return.[38] To simplify the analysis we assume that there are three possible hedging horizons: short, intermediate and long-term. Corresponding to each horizon, we respectively use the one-year, ten-year and thirty-year Treasuries. Thus, the size of the premium depends on two factors: 1) the hedging horizon and 2) the comparison of corporate average productivity for a given horizon versus the productivity trend(s) relevant to that horizon. In that respect, risk will manifest when the measure of productivity over the relevant horizon drops below the trend(s).

Recall from our previous section that while we measure the equity premium on a *nominal and after-tax basis*, this analysis preserves a constant after-tax real return in agreement with the Fisher/Darby/Feldstein effect, where *real* is defined according to the average inflation measure given the investor's hedging horizon. Over a thirty year horizon, because average productivity essentially tracks long-term trend, the business cycle premium is *zero*. In other words, according

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after-tax return. However, one possible explanation is that investors still aimed at earning the same minimum return, but anticipated greater future earnings than consensus estimates.

[36] This is for example the approach followed by Faugere and Van Erlach (2006).

[37] It is likely that investors attach some probability measure to this event. Although we do not integrate this feature in our analysis, our empirical results appear to validate our simpler approach.

[38] Fisher (2001) and Shen (2005) make the case that the marginal investor's horizon matters for the defining the risk premium. Obviously, Treasuries are not strictly risk-free. For example, investors still face coupon reinvestment risk in the case of a ten-year or thirty-year Treasury, and principal reinvestment (or price) risk in the case of three-month and six-month T-bills, to obtain the quoted annual yield. In our case, we assume that the correct hedging instrument is the Treasury that matches the investor's risk horizon.

to this definition of risk, the thirty-year Treasury should deliver a nominal after-tax yield equal to the nominal required yield.[39] In the intermediate term (ten-year horizon), business cycle risk still is defined with respect to long-term trend productivity. However in this case, the intermediate trend matters as well. For that horizon, we define the business cycle premium as the greater of two gaps: 1) the gap between long-term productivity vs. intermediate term productivity and 2) the gap between intermediate-trend productivity and itself, i.e. zero. Thus, in the intermediate term, the equity premium is strictly positive, because shorter horizons do not matter to that category of investors. Obviously, the risk premium is small in that case because intermediate term productivity tends to be close to long-term productivity anyhow.

In the short-run (one-year horizon), the business cycle premium is defined as the greater of two gaps: 1) the gap between intermediate-term productivity and *immediate* productivity vs. 2) the gap between short-term productivity (one-year) vs. immediate productivity. Investors with a short-term hedging horizon are concerned with the risk of being in an economic slowdown , or alternately, that the expected reversion of productivity to one-year and intermediate trends will not occur, when immediate productivity is currently below the short and intermediate-term trends. Why would the investor care about the intermediate trend in that case? Because, as we show later the intermediate horizon measure of productivity is also interpretable as another *one-year* forecast that accounts for the mean reversion of productivity to its long-run trend.

Notwithstanding, the premium may be *negative* with respect to short-term Treasuries. In that case, above normal immediate productivity induces investors to want to obtain a riskless rate that is greater than the required yield, but only in the short-run. One key reason is that (corporate) borrowers with short-term financing needs have the ability to pay this high return in the short-run due to immediate productivity spiking above future productivity. However, they are unlikely to sustain these payments over the long haul.

Let us denote by $g_{t+\Delta}$ the immediate nominal productivity, which is a simply the average of last-year's productivity and the one-year nominal forecast g_{t+1}^{E}. Let g_{t+1}^{10} represent a measure of the yearly average *nominal* expected productivity over the intermediate term (ten years), which is

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[39] We ignore the impact of the fear-based premium in this section. Prescott and McGrattan (2001) argue that the equity premium measured in reference to long-term debt has been close to zero on a tax-adjusted basis; due to what they view is a small systematic risk premium. Note that our argument stands in contrast with Bansal and Yaron's (2004)'s explanation, which relies on consumers being sensitive to nearly imperceptible fluctuations in long-run growth to accommodate the historical size of the equity premium. Our argument is that mean-reversion to a stable constant long-run real GDP/capita growth is what determines the valuation of assets. The fact that the government must pay a before-tax yield that outpaces economic productivity is possible if we assume that federal interest income taxes are in fact recycled into paying larger before-tax returns.

a weighted average of the one-year forecast of productivity g_{t+1}^{E} and long-term nominal GDP/capita growth, using the *average* mean reversion speed as a weight. Interestingly, g_{t+1}^{10} is also a forecast for *next-year's* productivity that accounts for the mean reversion of productivity to long-term GDP/capita growth.[40] The behavior of each of these measures of productivity (expressed in real terms) can be seen in Figure 4. We note that all these measures: immediate productivity, one-year forecast and the ten-year average appear to be non-stationary, with a structural break after 1990. Once again, the reason for this behavior is that our estimates are calculated based on forward earnings per share forecasts that were overly optimistic after the mid-1990s. Actual book value growth (not shown here) does not exhibit this same explosive behavior.

Figure 4 (About here)

Based on our above discussion, for a ten-year hedging horizon starting at time *t*, the business cycle premium is given by $B_{t+1}^{10} = Max\left\{g + \pi_{t+1}^{10} - g_{t+1}^{10}, 0\right\}$, where again *g* stands for the real long-term GDP/capita growth, the superscript stands for the hedging horizon, and the variable π_{t+1}^{10} represents the ten-year ahead average annual inflation forecast. In that case, the premium is positive when the ten-year expected average real yearly productivity is below long-term real GDP/capita growth. Alternately, this will hold when the economy is expected to slow down by comparison to a *one-year out* measure of productivity g_{t+1}^{10}, which accounts for mean reversion to long-run trend. Otherwise, when ten-year average productivity is greater than long-term GDP/capita growth, the premium is zero.

On the other hand, for a one-year horizon, we assert that the business cycle premium is defined by $B_{t+1}^{1} = Max\left\{g_{t+1}^{E} - g_{t+\Delta}, g_{t+1}^{10} - g_{t+\Delta}\right\}$. In that case, the premium is positive when the economy is in the midst of a down cycle, by comparison to the one-year forecast or the intermediate productivity trend. An interesting case is when immediate productivity is above both the ten-year trend and next year's forecast. In that case, the premium is *negative* and investors will demand a greater return than the required yield for the reasons indicated earlier.

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[40] Empirically; last year's productivity growth and g_{t+1}^{E} are essentially based on S&P 500 book value per share growth as described in Appendix A. We estimate an average mean reversion speed γ of 59%. Real is defined with respect to ten-years out expected inflation. In this case, our computation of the ten-year (intermediate) expected

6. Treasury Yields and Spread Determination on an After-Tax Basis

The goal of this section is to establish a theory of Treasury yield determination based on our business cycle risk approach to the equity premium and the required yield. From our analysis in Section 4, recall that the stock market required yield must be adjusted by including the fear-based premium as follows: $R^E_{t+1} = g + \pi^E_{t+1} + \Phi_{t+1}$, where $\Phi_{t+1} > 0$ stands for the fear-based risk premium.

Again, we denote by $r_{j\,t+1}$ the after-tax nominal expected yield on a bond with maturity *j = 1, 10, 30*. Paralleling the after-tax CAPM approach of Brennan (1970), we define the stock market required return in relation to the risk-free rate and the added risk premia as follows: $R^E_{t+1} + I^j_{t+1} = r^j_{t+1} + B^j_{t+1} + \lambda_j \Phi_{t+1}$.[41] The variable B^j_{t+1} represents the business cycle premium as defined in Section 5. The variable I^j_{t+1} represents the inflation risk premium for a Treasury of maturity *j*. To establish the after-tax CAPM, we add an inflation risk premium to the equity return R^E_{t+1}, to match the inflation risk premium that each Treasury yield already contains, as we argued previously. The parameter $\lambda_j = \lambda > 1$ represents the net effect of the fear-based premium, given that it is already contained in long term Treasury yields r^j_{t+1} for *j* =10, 30. The parameter for the one-year Treasury is set at $\lambda_1 = 1$ because we expect the fear-based risk to have zero net effect on the one-year Treasury, the reason being that investors prefer to lock in *long-term* rates when facing the prospect of an uncertain future catastrophic event.

After combining the above last two equations together, we obtain the crucial result of this section: $r^j_{t+1} = g + \pi^E_{t+1} + I^j_{t+1} - B^j_{t+1} - (\lambda_j - 1)\Phi_{t+1}$, for *j* = 1, 10, 30. When the fear-premium and inflation risk premia are absent, the stock market delivers the required yield, and Treasury yields are determined directly as a function of the required yield and the business cycle premium. When the fear-based premium is positive, it moves the stock market return and *long-term* Treasury returns symmetrically away from the required yield.

More importantly, it appears that Treasury yields are also compatible with the Fisher/Darby/Feldstein effect. Let the variable $\pi_{t+\Delta}$ represent the near-term inflation rate and we

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productivity assumes that the one-year expected productivity g^E_{t+1} is tracked for 10γ years (about 6.7 years) and long-run GDP/capita growth for 10(1- γ) years (about 3.3 years).

denote by upper-bar variables *real* rates after applying inflation rates (expected or actual) that correspond to the instrument's holding period.[42] After simple algebraic manipulations, and substituting the business cycle premia by their expressions, the above relationship for the one-year Treasury can be rewritten as a real yield $\overline{r}_{t+1}^{1} = \overline{g}_{t+\Delta} + Min\{g - \overline{g}_{t+1}^{10}, g - \overline{g}_{t+1}^{E}\}$ $+\pi_{t+\Delta} - \pi_{t+1}^{10} + I_{t+1}^{1}$. For the ten-year Treasury, the real yield is equal to $\overline{r}_{t+1}^{10} =$ $\overline{g}_{t+1}^{10} + Min\{g - \overline{g}_{t+1}^{10}, 0\} + \pi_{t+1}^{E} - \pi_{t+1}^{10} + I_{t+1}^{10} - (\lambda - 1)\Phi_{t+1}$.

Both the one-year and the ten-year treasuries after-tax real yields are determined in relation to real productivity corresponding to their respective horizon, a real business cycle premium that compares shorter-term real productivity to the long-term real GDP/capita growth, and inflation and fear-based premia. These real yields also depend on changes in the term structure of inflation. Interestingly, because the 10-year Treasury yield depends on $\overline{g}_{t+1}^{10}$, we can interpret the 10-year T-yield as a *one-year* expected yield that accounts for the mean reversion of productivity to long-term trend.

6.1. Accounting for the fear-based risk premium

An interesting by-product of the analysis is that we are able to estimate the size of the fear-based premium. We posited that the *business cycle* premium is zero for the thirty-year Treasury in comparison to the required yield. This implies that: $r_{t+1}^{30} = g + \pi_{t+1}^{E} + I_{t+1}^{30} - (\lambda - 1)\Phi_{t+1}$. Rewriting this equation, we get that the difference $g + \pi_{t+1}^{E} - r_{t+1}^{30} = (\lambda - 1)\Phi_{t+1} - I_{t+1}^{30}$, or that $\Phi_{t+1} = \dfrac{g + \pi_{t+1}^{E} - r_{t+1}^{30} + I_{t+1}^{30}}{\lambda - 1}$, which must be positive when there is a fear-based premium. Assuming that the fear-premium is fully symmetrical for Treasuries yields and the stock market return ($\lambda =$ 2), we get that the fear premium should only be applied when $\Phi_{t+1} = Max(g + \pi_{t+1}^{E} - r_{t+1}^{30} + I_{t+1}^{30}, 0)$; in other words, the (positive) fear premium is the difference between the nominal required yield and the after-tax nominal thirty-year Treasury yield, net of the inflation risk premium. Thus, periods when the after-tax thirty-year yield is below the

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[41] It is important to note that here we use the variable R_{t+1}^{E} which is different from the final required stock market return k_{t+1}^{E} used in our valuation formula, since the latter variable accounts for Treasury arbitrages, while the former does not.

[42] The only exception is that the real productivity measure g_{t+1}^{E} is based on the 10-year expected inflation rate.

nominal required yield should be a strong indicator that a fear-based premium is present.[43] When we incorporate the fear premium in our stock market valuation formula (8), we are able to fit the actual S&P 500 forward earnings yield very tightly after the second quarter of 2002, in contrast to the Fed model (Figure 3). This suggests that stock market valuation might have been under the influence of a fear premium from Q2 2002 till Q3 2006.

Figure 5 shows that the fear premium was positive during the 1994 Mexican Peso crisis, the 1995 Kobe earthquake, the 1997 Asian crisis and the 1998 Russian default crisis. After Q2 2000 the premium started steadily expanding till the end of our sample period in Q3 2006. During Q2 2003 to Q2 2004 the fear premium subsided. This corresponded to a bull market period. Note that prior to 1993, we do not observe any positive fear premia. One may have expected to find a positive fear premium during the 1981 recession, the 1987 market crash or even the 1991 recession. However, it is clear that at least in the case of the 1981 and 1991 recessions, these events were precipitated by economic policies that led to low economic forecasts. These economic risks should be contained in the business cycle risk premium. In fact, it is important that the fear premium be related to factors that are psychological in nature and not necessarily extractable from standard economic forecasts.

Figure 5 (About here)

6.2. A business cycle theory of the Treasury yield spread

Although not consistent over time and across countries, there is evidence that the yield spread constitutes a good predictor of real growth and of inflation at moderate horizons (Estrella and Mishkin, 1996; Kozicki, 1997). Past theories of the term structure (Fabozzi, 1997) indicate that the term premium is likely due to a liquidity premium (Liquidity Preference Hypothesis) or to a price risk premium (Expectations Theories), when the yield curve is normal. On the other hand, it becomes a reinvestment risk premium when the yield curve is inverted (Preferred-Habitat Theory). More recent models of the term structure have been based on no-arbitrage conditions and mean reverting interest rates processes embedded in dynamic term structure models (Vasicek, 1977; Heath, Morton and Jarrow, 1992) or generalized to the VAR methodology that incorporates other macroeconomic factors (Ang and Piazzesi, 2003).

[43] We need to apply caution in practice here. Our measure could possibly give wrong signals of positive fear premia in periods of high expected inflation, without being triggered by a "catastrophic" event. To avoid this possibility we trim the condition further and assume that the fear premium does only appear in periods where the expected inflation no higher than the historical average of about 3%.

Our theory is closer to the Preferred-Habitat Theory of Modigliani and Sutch (1966), in the sense that we posit that investors have preferred hedging horizons associated with investing in Treasuries, and they also select the maturity that is most able to maximize expected return within their hedging constraint. On the other hand, our theory also gives a foundation for determining these yields based on the productivity of equity capital, our modeling of the risk premium due to the business cycle, and the mean reversion of macroeconomic productivity.

So far, we have implicitly assumed that the fear-based premium is invariant with maturity (flat term-structure). Based on our expressions of Treasury yields from the previous section, we get:

$$\overline{r}_{t+1}^{10} - \overline{r}_{t+1}^{1} = TP_{t+1} = \overline{g}_{t+1}^{10} - \overline{g}_{t+\Delta}$$

$$+Min\{g - \overline{g}_{t+1}^{10}, 0\} - Min\{g - \overline{g}_{t+1}^{10}, g - \overline{g}_{t+1}\} + \pi_{t+1}^{E} - \pi_{t+\Delta} + I_{t+1}^{10} - I_{t+1}^{1} - (\lambda - 1)\Phi_{t+1} \quad (10)$$

Equation (10) is the core result of this section. It states that the *after-tax real* yield spread between the ten-year and one-year Treasuries (or term premium) TP_{t+1} is a function of real productivity growth differentials and expected inflation (and inflation risk) term structures, as well as the fear-based premium. This finding is consistent with the hypotheses put forth by Fama (1990) regarding the effect of the inflation risk premium and Mishkin's (1991) regarding the effect of real productivity of capital. The term premium TP_{t+1} is positive, when for example next year and intermediate term productivities are greater than near-term and long-term productivity. In that case, we are in an ascending phase of the business cycle and the economy is projected to grow faster in the intermediate term than in the near-term. For example this corresponds to the economic boom of the early 1990s; especially in 1994-1995 (see Figure 4).

In that case, ten-year Treasuries are exposed to a greater price risk than short-term bonds. The price risk comes from investors anticipating rising future spot rates and that selling the bond before maturity would yield a capital loss. In that case, rolling over one-year Treasuries appears to provide higher average returns with no adverse price risk. Thus, long-term investors bid up the price of one-year Treasuries, and bid down the price of ten-year treasuries, to compensate for the price premium. Short-term investors will let this process go forward, as long as they prefer a lower one-year T-yield to a higher yielding ten-year Treasury, which has an expected capital loss. Otherwise, they will prevent the price of the ten-year Treasury from sliding down further.

On the other hand, when the economy is entering a recession and next-year and intermediate productivities are less than long-term productivities with near-term productivity *greater* than intermediate productivity, the term premium is negative and represents a reinvestment risk premium. This corresponds for example to the economic situation of the 1974-75 recession (see Figure 4). The reinvestment risk is now smaller for long-term bonds. The risk is that future spot rates may drop over the next 10 years. Thus, it is preferable to lock in the average rate of the ten-year Treasury based on mean reversion, rather than risk rolling over one-year investments at potentially lower rates. Long-term investors will bid the price of ten-year Treasuries up and the one-year Treasury down.

This leads to an inverted yield curve (for these two maturities), on an after-tax nominal basis. Short-term investors will not counter this bidding behavior as long as they perceive that the expected capital gains from selling the ten-year Treasuries renders their total expected return larger than that of the one-year Treasury. Otherwise, they will not let the one-year Treasury price slide down as much.

The 2001 recession is an interesting case. The negative term premium is not directly explainable by the productivity differential, which does not indicate a slowdown; rather the yield curve inversion is explainable by a combination of the fear-based premium and the inflation risk premium.

Interestingly, monetary policy may also cause the behavior of the term premium. A sudden spike in expected inflation through the term $\pi_{t+1}^{E} - \pi_{t+\Delta}$ may result in a *positive* term premium, when the economy is on a path near its steady-state growth with near-term productivity close to its long-run average. On the other hand, under the same real growth conditions, when monetary authorities contract the money supply and inflation is expected to be reduced below its current value, a negative term premium (short-term yield curve inversion) may ensue. This last result is fully consistent with the current understanding of how the yield spread is correlated with *inflation* changes. However, our causality runs from monetary policy (inflation) to the yield spread, and not the other way around as may be found in the literature (Kocziki, 1997).

6.3. Empirical tests of Required Yield Theory for Treasury yields and the yield spread

The first period we examine is the whole sample period of Q4 1953- Q3 2006. The second period is Q4 1978- Q3 2006. The reason we focus on the post 1979 period is again that our estimates for Treasury yields incorporate productivity growth measures that are based on forward S&P 500 earnings, which are available only from 1979 on. For the same reasons mentioned before, empirical tests of our Treasury yield formulas are conducted on an after-tax and real basis, because these transformed variables are stationary.[44] All the regressions and discussion of methods used are presented in Appendix E.

We find that the adjusted R-squares from OLS regressions for all Treasuries are greater than 66% in all instances. The adjusted R-squares for the 10-year Treasury are 86% in each sample period, and 80% for the 30-year Treasury. Based on Newey-West estimations, all slope coefficients are significant above the 1% level and are closer to unity for the period 1979-2006. Figures 6-a, -b and -c illustrate how well our model fits the Treasury data graphically.

The yield spread is well-explained by our theory with an R-squared of 58% in the main sample and of 69% in the sub-period Q4 1978- Q3 2006. Interestingly, the slope coefficient remains slightly under unity for both sample periods. The reason seems due to the dynamics of our estimates prior to 1979. As we use ex-post next year's earnings per-share prior to 1979 to calculate book value growth rates, we are underestimating the magnitude of positive spreads due to the market having overly optimistic forecasts (Figure 6d).

We also observe that prior to 1979 the actual yield spread leads our own estimate by about two quarters. This means that the market appears to anticipate by about two quarters in advance the actual level of future S&P 500 earnings. In that respect, the yield spread can be viewed as a leading indicator of future levels of S&P 500 earnings. After 1979, we observe that the two curves are much more synchronized, which implies that the yield spread incorporates estimates that are close to analysts' forecasts. More importantly, looking at Figure 6d we appear to successfully track historical yield curve inversions. Out of twelve inversions over the entire period, we accurately replicate ten of them, with one false signal.

Table 4 (About here)

Figures 6 a, b, c, d (About here)

7. Implications: The Inflation Illusion Hypothesis, the Fed Model and Ex-Post Equity Premium

7.1. The Fisher effect and inflation illusion

Over the last several decades, empirical tests of the strict Fisher effect (absent of tax considerations) have shown mixed results. *Ex-post* stock returns appear to be negatively related to inflation in the short-run (Bodie, 1976; Nelson, 1976). Although, these findings do not strictly invalidate the Fisher hypothesis, they obviously do not confirm this hypothesis either. Ex-post

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[44] These results are not shown here, but are available from the authors upon request.

returns, for example, may serve as good indicators of ex-ante returns only under very strict conditions about the behavior of stock prices. What is needed is a tall order: the absence of information surprises about fundamentals when measuring end-of-period prices (Elton, 1999). On the other hand, Boudoukh and Richardson (1993) show that the Fisher effect seems to hold in the long-run. Many economists have puzzled over the fact that stocks appear not to provide an inflation hedge, as economic theory would predict. Several alternative theories have been suggested to explain that counterintuitive finding (Feldstein, 1980; Fama, 1981; and Geske and Roll, 1983).

On the other hand, while strictly *not* being in violation of the Fisher/Darby/Feldstein effect, there is a large body of evidence showing that the stock market P-E is *negatively* affected by inflation expectations (Reilly, Griggs and Wong, 1983; Modigliani and Cohn, 1979; Sharpe, 1999, 2001). Some economists argue that the inverse P-E (earnings yield) is a 'real' quantum and should not therefore, be affected by inflation expectations (Ritter and Warr, 2002; Siegel, 2002). Several academics have invoked an *inflation illusion* argument to explain this apparent paradox (Modigliani and Cohn, 1979; Ritter and Warr, 2002; and Campbell and Vuolteenaho, 2004).

In their analysis of this phenomenon, Modigliani and Cohn (1979), suggest that investors "are plagued by a form of money illusion". Investors appear to use nominal discount rates to discount 'real' earnings, and/or they fail to recognize the fact that inflation reduces the real value of the principal owed. Ritter and Warr (2002) are in the same camp and conclude that equities were undervalued into the early 1980s because of "cognitive valuation errors of levered stocks in the presence of inflation and mistakes in the use of nominal and real capitalization rates." They credit the subsequent Bull market of 1982 to 1999 in part due to a falling risk premium.

The core argument from Modigliani and Cohn (1979) is that since corporate earnings are indexed at the rate of actual inflation, the value of a stock should be unaffected by changes in the inflation rate as the capitalization rate remains constant. A simple illustration of their argument is to use the Gordon growth model (Campbell and Vuolteenaho, 2004). The Gordon growth model states that in a steady-state, the price of a stock is given by the value of a growing perpetuity of dividends. Using our notations,

$$P_t = \frac{D_{t+1}^E}{k-g} = \frac{D_{t+1}^E}{\tilde{k}-\tilde{g}} \tag{11}$$

Where k and g respectively are the long-term required return and growth rate of dividends, and $\tilde{k}$ and $\tilde{g}$ are their real counterparts. In that case, the price of the stock is unaffected whether rates are in nominal or real terms, since the inflation term cancels out in the denominator, and the capitalization rate $(k-g)$ remains unchanged.[45] Upon closer examination, however, this argument makes the implicit assumption that a shift in the expected inflation rate results in a *same* magnitude change for both the required return and the growth rate. However, this may not always be the case, for example when taxes distort real returns (Darby, 1975; Feldstein, 1976). Another case is that of S&P 500 companies having zero growth opportunities in the long run.

Take the case of an index (like the S&P500) where the nominal sustainable growth rate is given by the reinvestment hypothesis; i.e. $g = \tilde{g} + \pi = (1-b)k$, where the variable (1-*b*) stands for the retention ratio. In the long-run, expected productivity equals the long-run growth rate of GDP/capita. In other words, due to zero growth opportunities, the stock market nominal return satisfies: $k = \dfrac{\tilde{g}+\pi}{(1-b)}$. After substituting the expression for k in the first part of equation (11), we get:[46]

$$P_t = \frac{D_{t+1}^E}{k-g} = \frac{be_{t+1}^E}{\dfrac{\tilde{g}+\pi}{(1-b)} - \tilde{g}+\pi} = \frac{(1-b)e_{t+1}^E}{\tilde{g}+\pi} \tag{12}$$

This means that when nominal growth opportunities are zero in the long-run, the stock price and the P-E ratio *should be* negatively related to the long-run inflation rate.[47] On the other hand, for Modigliani and Cohn's (1979) argument to hold, it must be the case that $\tilde{k} = \dfrac{\tilde{g}}{(1-b)}$. This means that the real return would be given directly by the real sustainable growth rate, where the retention ratio applies to *real* earnings. Thus, inflation would have the same one-to-one effect on the nominal return and growth rate.

However, in the steady state, nominal growth of aggregate earnings per share must follow the nominal growth rate of GDP/capita, which must be achieved via retaining *nominal* earnings. Thus, the relationship $k = \dfrac{\tilde{g}+\pi}{(1-b)}$ must be true in the long run. Faugere and Van Erlach (2006) show theoretically and empirically that the S&P500 historical market average return is fully

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[45] This result occurs due to the approximation $k = \tilde{k} + \pi$. On the other hand, when we use the *correct* Fisher hypothesis: $(1+k) = (1+\tilde{k})(1+\pi)$, the right conclusion is that the stock price deflated by the general price index, is independent of inflation.

[46] Note that if we assume that the required return is before-tax, we would get back our valuation formula (9), as long as the effective tax rate equals the payout ratio in the steady-state, which is a long-run constraint for our theory.

[47] Caution must be exercised in drawing this conclusion, since it is a steady-state relationship. A shift in the long-run rate of inflation would take time to be reflected in the long-run sustainable growth rate, since the real growth rate would drop in the short to medium run, as we explained earlier.

explained with this last formula, using as inputs historical average GDP/capita growth, average payout ratio and inflation rate over the period 1926-2001.

Outside steady-state considerations, Campbell and Vuolteenaho (2004) discuss a variation of the inflation illusion argument: that investors incorrectly extrapolate past nominal growth forecasts, without considering future changes in inflation. Sharpe (2001) argues the same point about analysts' forecasts. However, even though analysts' forecasts are overly optimistic in levels, they also exhibit mean reversion to the *same* growth trend as actual earnings.

Our theory does not rely on an inflation illusion explanation. Our approach allows for time varying expected discount rates and expected inflation, while only stipulating that growth opportunities are mean reverting to zero in nominal terms. This does not entail the presence of any biases in inflation or nominal discount rates expectations, as long as the speed of mean reversion properly adjusts to changes in expected inflation. In other words, investors should realize that a boost in expected inflation will *slow down* the speed of mean reversion and vice versa. To that extent, our model is restrictive by assuming that the mean reversion speed is essentially independent of inflation expectations.

Sharpe (1999, 2001) finds that high expected inflation predicts low stock returns (and high dividend yields) and has a strong negative correlation to the P-E. In his 2001 article, he concludes that this effect coincides with either lower expected real earnings growth and/or higher required real returns. Our theory indeed demonstrates that the effect of an anticipated rise of inflation is to reduce the real sustainable growth rate, which reduces the real return only in the case of an up cycle. However, investors still wish to preserve the required yield, which is at least equal to the real long-term GDP/capita growth rate. Thus, in the case where inflation rises and the economy is in a down cycle, the real return would remain at the long-term GDP/capita growth level. Thus, our theory is together consistent with Sharpe's observation and the Fisher/Darby/Feldstein effect.[48]

7.2. Required Yield Theory and the Fed model

Our Required Yield Theory establishes a firm theoretical foundation for why the Fed model (Lander, Orphanides et al. (1997)) appears to work empirically. Our theory encompasses the Fed model to account for the impact of taxes as well as inflation, and the investors' requirement to obtain an after-tax real return pegged to the long run real GDP/capita growth rate. Ritter (2001)

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[48] In that sense, our result does not contradict the findings that *ex-post* returns do not satisfy the Fisher effect in the short-run.

notes that the Fed model works better empirically than other models, but should not work well theoretically if most of the variation in nominal rates and thus stock yields comes from changes in expected inflation rather than changes in real rates.

The logic is that for the earnings yield to move one-for-one with the nominal bond yield, as the in Fed model, one has to assume that the nominal yield on bonds equals the real return on stocks, since the earnings yield is *viewed* as a real return. Thus, the empirical success of the Fed model appears to be inconsistent with rational valuation according to current theory. Whereas it is true that contemporaneous earnings divided by current stock price is a real quantum, because both quantities are deflated by the general price level, and also that nominal earnings are indexed with inflation, it does not follow that the current earnings yield is a *real* yield in the sense of the Fisher relation, since it is missing the effect of inflation on the *principal* invested. At any rate, the trailing earnings yield is not an appropriate measure of expected return. A better measure is the forward earnings yield.

According to Required Yield Theory the forward earnings yield is determined in relation to a nominal return that is built up from the requirement that investors receive a minimum after-tax expected real return on stocks at all times at least equal to long-term GDP/capita growth. Asness (2003) argues that the Fed model does not provide an absolute benchmark for valuing stocks. Here we show that the yields on long-term treasuries are related to the market inverse P-E, since our approach demonstrates that both are related to a third common factor, our hitherto defined required yield.

7.3. Why are ex-post market returns and the equity premium so large?

Some readers may still be puzzled as to why we argue that the stock market has essentially returned a real 2.21% after-tax, given that ex-post real after-tax returns appear to have vastly exceeded 2.21%. As seen in our first section on unit root tests, we estimated the mean reversion constant for the after-tax real total S&P 500 return at a value of 5.23%. Our theory, however, is able to account for this discrepancy. Let us go back to our stock market valuation formula (8) and assume zero abnormal earnings growth. The before-tax ex-post market total return is given by:

$$R_{t+1}^{E} = \frac{D_{t+1}}{P_t} + \frac{(P_{t+1} - P_t)}{P_t} = \frac{D_{t+1}}{P_t} + \left[\frac{e_{t+2}^{E}}{e_{t+1}^{E}} \times \frac{(1-\tau_{t+2}) \times (g + \pi_{t+1}^{E})}{(1-\tau_{t+1}) \times (g + \pi_{t+2}^{E})} \right] - 1 \qquad (13)$$

Equation (13) shows how it is possible to have witnessed such a large average ex-post equity return over the last fifty years. Essentially, the second term on the right hand side indicates that for a given rate of EPS earnings growth close to GDP/capita growth, the historical combination of declining blended tax rates and declining inflation expectations is the main reason that *ex-post* capital gains were abnormally *high*.

Furthermore, the dividend yield has also been affected by tax trends. Given that the *relative* dividend income tax rate has declined in comparison to the capital gains tax rate by about 1.96% per year on average, over the period Q4 1953- Q3 2006. This effectively boosted the dividend yield by that same amount to its historical average of 3.73% (calculated based on one-year forward expected earnings). Firms still were able to *increase* their nominal dividend payouts without sacrificing sustainable growth, assuming that dividends were substituted for share repurchases in the same proportion as the relative tax change.[49]

These effects on ex-post capital gains rate and dividend yields may contribute to the empirical findings that ex-post returns have been found to vary inversely with *changes* in expected inflation rates (Bodie, 1976; Nelson, 1976), since trends in inflation and blended tax rates (as well as dividend tax rates relative to capital gains) were all correlated over the period.

Notwithstanding, in the absence of such trends the average stock market return would have been capped by the (adjusted dividend yield) + EPS growth. The adjusted dividend yield is (3.73% −1.96%). EPS growth equals long-term nominal GDP/capita growth at about 5.61% = 2.03% + 3.58% (inflation). In other words, the average return would have been about (3.73% −1.96%) + 5.61% = 7.4% versus the 11.6% *compounded* average that was observed over the period Q4 1953- Q3 2006. Interestingly, our adjusted estimate is very close to the average forward earnings yield over the period, which has been 7.7%. The fact that the ex-post total compound return minus inflation turns out to be 8.0%, which is also close to the forward earnings yield, is the main reason why observers (Ritter, 2001; Siegel, 2002; Asness, 2003; Campbell and Vuolteenaho, 2004) generally believe that the (forward) earnings yield is a *real* return.

However, it is a well-known result (Reilly and Brown, 2006) that the forward earnings yield is equal to the total return when the present value of growth opportunities is zero. Our result indicates that we can view the earnings yield as the total *nominal return*, would the S&P 500 not have produced these *abnormal positive returns* due to falling tax rates and inflation rates. Hence, the size of the ex-post equity premium (difference between ex-post market return minus a

[49] Furthermore, while there was a spike in the top marginal corporate tax rate in 1984-1986, the rest of the time from 1971-2002, the general trend was a decrease in that rate, which may have boosted free cash flows. Sources: U.S.

Treasury yield) is also explainable in light of the drivers of the large historical ex-post returns: downward trending taxes in absolute and relative terms (favoring dividends), and a steady decline of actual and expected inflation.

8. Seven Outstanding Puzzles of Finance Revisited

Having examined the implications Required Yield Theory for stock market valuation, Treasury yield determination and the various sources of the equity premium, we are now ready to propose a comprehensive resolution of seven major outstanding puzzles of Finance:

Puzzle #1: Why are stock market prices more volatile than expected dividends?

Our answer: Shiller (1981) argues that if stocks prices are determined by the present value of expected dividends, prices should fluctuate in the same manner as expected dividends. However, he finds that this property is violated in the data, i.e. prices are more volatile than dividends. This puts into question the validity of the standard discount valuation model. Although Shiller (1981) assumes that both stock prices and dividends are stationary around a stochastic trend, Campbell and Shiller (1988a, 1988b) find evidence for excess volatility even allowing for unit roots. Cochrane (1992) claims that the puzzle can be explained by innovations in excess returns. Barsky and Delong (1993) alternately argue that changes in the permanent component of dividend growth can also explain the puzzle. Balke and Wohar (2002) agree that these two explanations are likely candidates but their tests cannot determine which alternative prevails.

Required Yield Theory shows that the market is indeed priced as the present value of expected dividends, with the proviso that rational investors expect the present value of nominal growth opportunities to mean revert to zero quickly. Ex-post dividend fluctuations do not play a major role in explaining market price fluctuations, as Shiller (1981) finds. Rather, at low frequency (quarterly data) we find that S&P 500 prices fluctuate mostly in response to changes in expected earnings, inflation rates and marginal tax rates. The latter two variables directly impact the required return on equity via the Fisher/Darby/Feldstein effect. Because investors appear to price the index based on wanting to earn a real return equal to a *constant* real long-term GDP/capita growth after-tax, we do not need to appeal to permanent changes in dividend growth as an explanation for stock market volatility.

House of Representatives, U.S. Congress, 'The Overview of the Federal Tax System", 102d Congress (Washington, DC: U.S. Government Printing Office, April 10, 1991) and Devereux, Griffith and Klemm (2002).

Based on our model, the calculated yearly standard deviation of S&P 500 total returns over the period is 26% versus 27% actual.[50] On a quarterly basis, our formula tracks the S&P 500 earnings yield with a level of accuracy superior to the Fed Model (Lander at al, 1997) at an adjusted R-squared of 88% for 1954-2006 and 94% for 1979-2006.

Puzzle #2: Why is the equity premium not accounted for by standard measures of risk and our best asset pricing models?

Our answer: we define the ex-ante after-tax equity premium as the difference between the nominal required yield and the after-tax yield on a Treasury (for any given maturity). We show that the equity premium is mostly explained by business cycle risk; i.e. the risk that mean reversion of productivity growth will not catch-up to its anticipated level. This approach enables us to closely match the actual behavior of after-tax Treasury yields as a function of the after-tax expected equity return (nominal required yield) and short-term productivity measures (sustainable growth rate of corporate earnings per share). (See puzzle #4 below).

Inflation and fear-based risks play a secondary role in the determination of the equity premium. We are able to characterize the fear-based risk premium based on departures of the after-tax 30-year yield when it drops below the required yield. The argument is that these departures reflect flight to safety, given that the equity premium over long-term bonds should be zero in the absence of fear-based and inflation risk. We explain the large size of the ex-post before-tax nominal premium, by showing that ex-post returns did inflate due to a decline in the rate of inflation and downward trending tax rates in absolute and relative terms (favoring dividends).

Puzzle #3: Why aren't stocks behaving as an inflation hedge instrument, as common sense would dictate?

Our answer: In agreement with the literature, we find that stock market prices are indeed *not* indexed with expected inflation. However, expected stock returns satisfy the Fisher/Darby/Feldstein effect. In other words, investors seek protection against increases in expected inflation and taxes.

Puzzle #4: Why aren't stock returns and Treasury yields more directly connected to measures of productivity/economic growth?

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[50] We compute total annual return (every quarter) as annualized (x4) quarterly capital gains rate plus annualized quarterly dividend divided by stock price 4 quarters earlier. The annual standard deviation is calculated based on Q4 to Q4 return observations.

Our answer: We show that both the U.S. stock market return and Treasury yields are functions of long term U.S. GDP/capita growth. Short term treasury yields are also a function of accelerations or deceleration of short-tem productivity growth relative to long-term trends. Empirically we track the yields on the 1-year, 10-year and 30-year Treasuries with adjusted R-squares over 66% in all cases.

Puzzle #5: Why does the yield spread appear to be a good predictor of real economic cycles?

Our answer: The reason is actually simple. The yield spread is already a function of productivity growth differentials between short term expectations vs. longer term trends. These productivity growth expectations embed corporate earnings forecasts. We can empirically track the yield spread between the 10-year and the 1-year Treasury over the last fifty years with an adjusted R-squared of 58% and an R-squared of 69% over the period 1979-2006. More importantly, we are able to reproduce ten out of twelve yield curve inversions over the whole sample period, with only one false signal.

Puzzle #6: Why do Treasury yields and the stock market earnings yield appear to behave as non-mean reverting processes when stock market returns are found to be mean-reverting in some instances?

Our answer: Our unit root tests show that after-tax real yields (Treasuries and stock market total return and forward earnings yield) are stationary processes (at the 1% significance level), over the period Q4 1953- Q3 2006 , which is a novel finding in the literature.

Puzzle #7: Why is the so-called Fed model, which links government bond yields and P-E ratios of market indexes, found to be a global empirical regularity, in spite of its perceived logical flaws?

Our answer: The yields on long-term U.S. treasuries and stock market are both tied to long-term real U.S. GDP/capita growth (the required yield). The forward earnings yield is a function of the 10-year Treasury yield when it provides an excess yield over the required yield, i.e. in periods of high inflation risk premia. Our RYT stock market valuation formula (8) validates and generalizes the Fed model.

9. Conclusion

In this article, we have introduced a general theory for valuing a broad market index (S&P 500) and for determining the yield on long-term (thirty-year and ten-year) and short-term (one-year) Treasuries. Required Yield Theory is founded on the Fisher effect (1896) as generalized by Darby (1975) and Feldstein (1976). A series of open issues remain for future investigation.

While we account for the term structure of expected inflation via its effect on the risk premium, it is somewhat puzzling that our theory based on the Fisher/Darby/Feldstein effect should describe well the behavior of the stock market forward earnings yield and long-term Treasury yields based mostly on *one-year* inflation forecasts. In other words, how much are our results an artifact of a monetary policy that credibly sustains low inflation in the short- and long-term?

Another issue is that our theory while grounded in the concept of GDP/capita growth as the minimum return, does not rule out competing explanations. As the market share of funds offering tax-deferred vehicles has grown over the last four decades, it may be interesting to analyze how this shift may have affected the effective tax rate of the marginal investor. For example, an excess premium in addition to the 2.21% long-term GDP/capita growth rate may be possible by holding a tax-deferred account, if the marginal investor has a higher effective tax rate than the person benefiting from deferred marginal rates.

Our model can also be extended by making the mean reversion speed of PVGOs dependent on inflation expectations, as investors should realize that higher inflation will *slow down* the speed of mean reversion and vice versa. The fact the market participants do not seem to correct for this effect when pricing the index, may constitute a lesser form of inflation illusion. Finally, our theory applied to the yield spread indicates that the spread may have predictive power regarding the level of future S&P 500 earnings per share. This last result merits further investigation.

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Figure 1: After-Tax Real S&P 500 Forward Earnings Yield and 10-Year Treasury Yield. Q4 1953- Q3 2006.

Table 1: Unit Root Tests for the 1-year, 10-year Treasuries and the S&P 500 Total Return and Earnings Yield

The null hypothesis is that these time series are non-stationary (Slope d = 0). The testing period is Q4-1953 to Q3-2006. The Akaike and Bayesian information criteria for goodness of fit are shown on the table as well as the critical values for the ADF test at the 5% and 1% levels of significance.

		Nominal Yields				Real After-Tax Yields				Critical
# Lags		1-year T	10-year T	Market Return	Market E-P	1-year T	10-year T	Market Return	Market E-P	Values
Panel A: Augmented Dickey-Fuller Baseline Tests										
4	Slope δ	-0.90% (-0.84)	-0.31% (-0.53)	-61.41% (-5.14)	-0.43% (-0.71)	-5.94% (-1.96)	-2.36% (-1.47)	-80.32% (-6.03)	-2.00% (-1.47)	1.95; 2.59
	AIC; BIC	-1313; -1296	-1520; -1503	-128; -145	-1457; -1441	-1471; -1454	-1648; -1632	3; 20	-1628; -1612	
3	Slope δ	-0.85% (-0.80)	-0.32% (-0.54)	-62.30% (-5.61)	-0.46% (-0.77)	-5.95% (-1.99)	-2.28% (-1.43)	-77.95% (-6.43)	-2.22% (-1.65)	1.95; 2.59
	AIC; BIC	-1322; -1308	-1530; -1516	-128; -141	-1466; -1453	-1481; -1468	-1659; -1646	2; 15	-1638; -1624	
2	Slope δ	-0.70% (-0.65)	-0.26% (-0.45)	-67.75% (-6.71)	-0.51% (-0.85)	-5.91% (-2.00)	-2.15% (-1.36)	-80.84% (-7.52)	-2.54% (-1.9)	1.95; 2.59
	AIC; BIC	-1327; -1317	-1534; -1524	-129; -139	-1476; -1466	-1491; -1481	-1668; -1658	1; 11	-1645; -1634	
1	Slope δ	-0.90% (-0.84)	-0.29% (-0.49)	-74.07% (-8.52)	-0.54% (-0.92)	-7.95% (-2.68)	-2.52% (-1.59)	-83.57% (-9.26)	-2.80% (-2.12)	1.95; 2.59
	AIC; BIC	-1330; -1323	-1544; -1538	-129; -136	-1485; -1479	-1488; -1481	-1675; -1668	-1; 6	-1652; -1646	
0	Slope δ	-1.11% (-1.03)	-0.31% (-0.53)	-79.60% (-11.80)	-0.55% (-0.95)	-9.43% (-3.23)	-2.78% (-1.77)	-85.31% (-12.51)	-2.97% (-2.29)	1.95; 2.59
	AIC; BIC	-1334; -1331	-1553; -1550	-129; -132	-1493; -1489	-1493; -1490	-1684; -1681	-2; 1	-1662; -1659	
Panel B: Augmented Dickey-Fuller Tests with Drift										
4	Slope δ	-6.20% (-2.57)	-3.19% (-2.01)	-102.05% (-6.80)	-3.43% (-1.87)	-20.64% (-3.87)	-13.12% (-3.59)	-94.23% (-6.6)	-14.87% (-3.75)	-1.65; -2.35
	Drift β_0	0.38% (2.44)	0.22% (1.95)	11.97% (4.21)	0.26% (1.73)	0.27% (3.32)	0.23% (3.26)	4.37% (2.42)	0.33% (3.45)	
	AIC; BIC	-1317; -1297	-1521; -1501	-113; -133	-1458; -1438	-1480; -1460	-1657; -1637	-1, 19	-1639; -1619	
3	Slope δ	-5.90% (-2.47)	-3.05% (-1.95)	-96.25% (-7.19)	-3.60% (-1.98)	-19.50% (-3.80)	-11.90% (-3.35)	-89.43% (-6.93)	-15.22% (-4.02)	-1.65; -2.35
	Drift β_0	0.36% (2.36)	0.21% (1.88)	11.44% (4.20)	0.27% (1.83)	0.26% (3.21)	0.20% (3.02)	4.26% (2.40)	0.33% (3.66)	
	AIC; BIC	-1325; -1309	-1531; -1515	-112; -129	-1468; -1451	-1489; -1473	-1666; -1650	-2; 14	-1650; -1633	
2	Slope δ	-5.21% (-2.20)	-2.62% (-1.67)	-96.25% (-8.33)	-3.67% (-2.05)	-18.21% (-3.68)	-10.43% (-2.99)	-90.37% (-8.04)	-16.24% (-4.52)	-1.65; -2.35
	Drift β_0	0.32% (2.14)	0.18% (1.62)	11.61% (4.49)	0.27% (1.87)	0.24% (3.07)	0.18% (2.65)	4.45% (2.56)	0.36% (4.09)	
	AIC; BIC	-1329; -1315	-1535; -1521	-111; -124	-1477; -1464	-1498; -1485	-1673; -1660	-4; 10	-1659; -1646	
1	Slope δ	-5.74% (-2.44)	-2.63% (-1.69)	-95.41% (-10.14)	-3.76% (-2.13)	-22.35% (-4.66)	-11.57% (-3.41)	-910.58% (-9.77)	-16.69% (-4.93)	-1.65; -2.35
	Drift β_0	0.35% (2.31)	0.18% (1.63)	11.64% (4.76)	0.28% (1.93)	0.29% (3.75)	0.19% (3.00)	4.56% (2.68)	0.37% (4.42)	
	AIC; BIC	-1333; -1323	-1545; -1535	-109; -119	-1487; -1477	-1500; -1490	-1682; -1671	-6; 4	-1669; -1659	
0	Slope δ	-6.24% (-2.69)	-2.80% (-1.83)	-92.18% (-13.38)	-3.45% (-1.96)	-24.07% (-5.35)	-11.87% (-3.62)	-89.38% (-13.01)	-15.73% (-4.92)	-1.65; -2.35
	Drift β_0	0.37% (2.49)	0.19% (1.76)	11.36% (4.93)	0.25% (1.75)	0.30% (4.16)	0.20% (3.13)	4.60% (2.77)	0.34% (4.33)	
	AIC; BIC	-1339; -1332	-1554; -1548	-108; -114	-1494; -1487	-1508; -1501	-1692; -1685	-8; -1	-1678; -1671	
Panel C: Augmented Dickey-Fuller Tests with Drift and Trend										
4	Slope δ	-6.07% (-2.49)	-2.85% (-1.68)	-102.00% (-6.78)	-3.40% (-1.86)	-22.46% (-3.98)	-13.59% (-3.68)	-94.25% (-6.55)	-14.87% (-3.61)	-3.44; -4.01
	Drift β_0	0.43% (2.24)	0.25% (2.03)	11.43% (2.32)	0.27% (1.52)	0.42% (3.15)	0.29% (3.03)	3.10% (0.89)	0.33% (2.56)	
	Trend β_1	0.00% (-0.42)	0.00% (-0.59)	0.00% (-0.23)	0.00% (-0.10)	0.00% (-1.37)	0.00% (-0.92)	0.00% (0.42)	0.00% (-0.01)	
	AIC; BIC	-1315; -1292	-1520; -1496	-115; -138	-1456; -1433	-1480; -1457	-1656; -1633	1; 24	-1637; -1613	
3	Slope δ	-5.72% (-2.39)	-2.75% (-1.65)	-96.25% (-7.17)	-3.60% (-1.98)	-21.01% (-3.99)	-12.31% (-3.43)	-89.41% (-6.91)	-15.23% (-3.87)	-3.44; -4.01
	Drift β_0	0.41% (2.21)	0.23% (1.94)	11.42% (2.37)	0.27% (1.58)	0.38% (2.99)	0.26% (2.80)	3.45% (1.00)	0.33% (2.72)	
	Trend β_1	0.00% (-0.49)	0.00% (-0.54)	0.00% (0.01)	0.00% (-0.05)	0.00% (-1.25)	0.00% (-0.85)	0.00% (0.27)	0.00% (-0.02)	
	AIC; BIC	-1324; -1304	-1530; -1510	-114; -134	-1466; -1446	-1489; -1469	-1665; -1645	0; 19	-1647; -1627	
2	Slope δ	-5.01% (-2.10)	-2.19% (-1.31)	-96.31% (-8.31)	-3.67% (-2.04)	-19.50% (-3.84)	-10.77% (-3.06)	-90.35% (-8.02)	-16.38% (-4.39)	-3.44; -4.01
	Drift β_0	0.39% (2.12)	0.21% (1.79)	12.13% (2.59)	0.27% (1.57)	0.35% (2.79)	0.22% (2.44)	4.08% (1.20)	0.37% (3.10)	
	Trend β_1	0.00% (-0.63)	0.00% (-0.77)	0.00% (-0.13)	0.00% (0.04)	0.00% (-1.10)	0.00% (-0.72)	0.00% (0.13)	0.00% (-0.14)	
	AIC; BIC	-1328; -1311	-1534; -1517	-113; -130	-1475; -1458	-1498; -1481	-1671; -1655	-2; 15	-1657; -1641	
1	Slope δ	-5.60% (-2.36)	-2.28% (-1.38)	-95.49% (-10.11)	-3.75% (-2.12)	-23.46% (-4.79)	-11.87% (-3.46)	-90.58% (-9.75)	-16.93% (-4.82)	-3.44; -4.01
	Drift β_0	0.39% (2.13)	0.21% (1.75)	12.51% (2.75)	0.27% (1.58)	0.40% (3.23)	0.24% (2.65)	4.51% (1.34)	0.39% (3.42)	
	Trend β_1	0.00% (-0.41)	0.00% (-0.65)	0.01% (-0.23)	0.00% (0.11)	0.00% (-1.13)	0.00% (-0.67)	0.00% (0.02)	0.00% (-0.28)	
	AIC; BIC	-1332; -1318	-1540; -1530	-111; -125	-1485; -1471	-1499; -1486	-1680; -1667	-4; 10	-1667; -1654	
0	Slope δ	-6.18% (-2.63)	-2.50% (-1.54)	-92.25% (-13.35)	-3.43% (-1.95)	-25.00% (-5.45)	-12.11% (-3.65)	-89.40% (-12.97)	-15.81% (-4.76)	-3.44; -4.01
	Drift β_0	0.40% (2.17)	0.21% (1.84)	12.55% (2.84)	0.23% (1.37)	0.41% (3.43)	0.23% (2.67)	4.85% (1.47)	0.35% (3.19)	
	Trend β_1	0.00% (-0.21)	0.00% (-0.56)	0.01% (-0.32)	0.00% (0.22)	0.00% (-1.07)	0.00% (-0.57)	0.00% (-0.09)	0.00% (-0.08)	
	AIC; BIC	-1337; -1327	-1553; -1543	-110; -120	-1492; -1482	-1507; -1497	-1690; -1680	-6; 4	-1676; -1666	

Figure 2: S&P 500 Present Value of Growth Opportunities vs. NBER Recessions. Q4 1953- Q3 2006. Recessions are in shaded area.

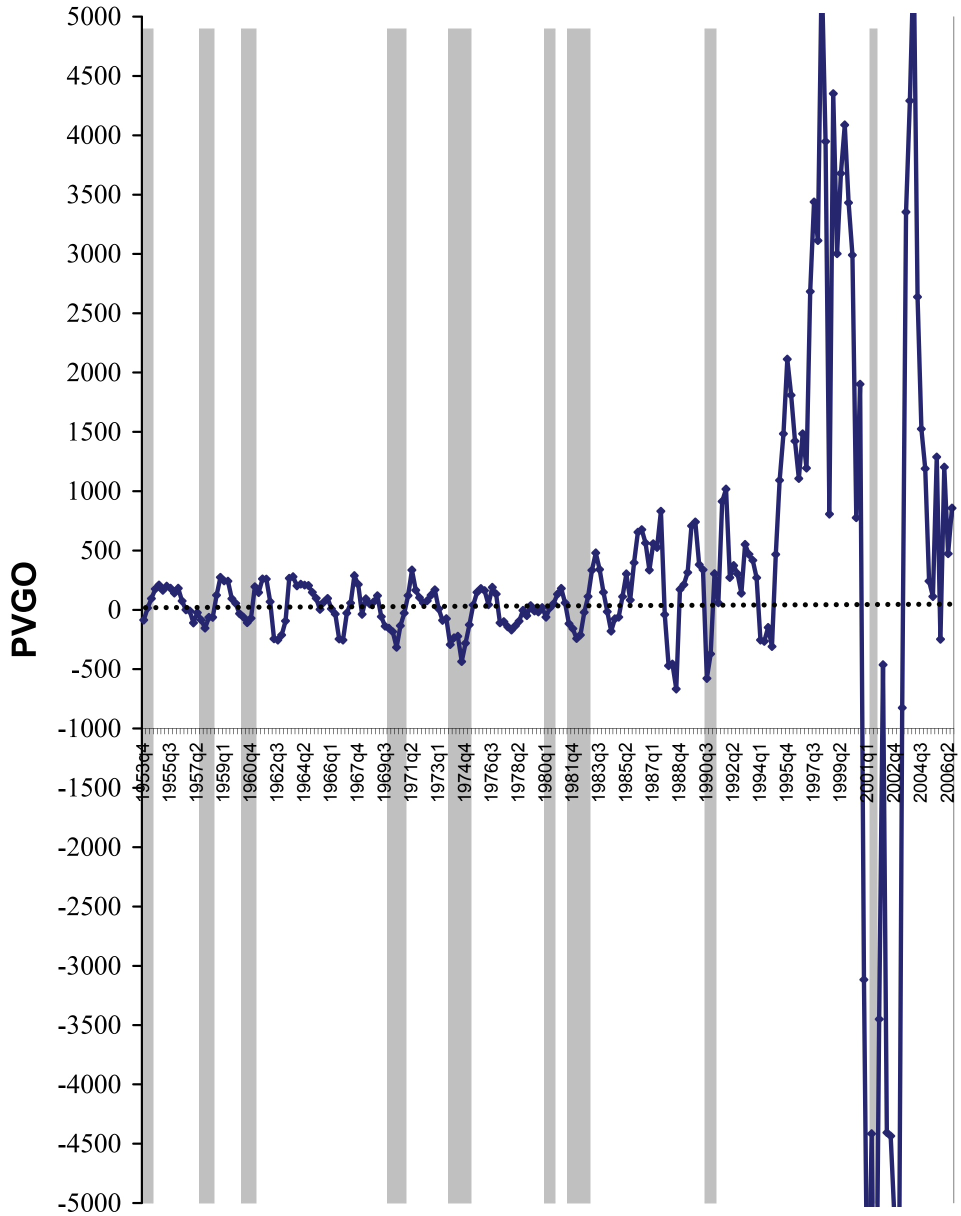


Table 2: Real GDP per Capita Growth 1929-2006

GDP Growth data is from the St. Louis FED and population data is from the US Bureau of Census.

Period	1929-2001	1929-2006	1954-2001	1954-2006
Real GDP growth	3.56%	3.41%	3.56%	3.34%
Population growth	1.32%	1.38%	1.31%	1.40%
Real GDP/capita growth	2.24%	2.03%	2.25%	1.94%

Table 3: Estimation of After-Tax Real S&P 500 Forward Earnings Yield Using RYT vs. Fed Model

We test the RYT regressions: $ey^E_{i\,t+1} = \beta_1 \times r^E_{t+1} + \beta_2 \times \pi^E_{t+1} + v_{i\,t+1}$ $i = 1, 2$ for each reversion speed i from above or below

We test the Fed Model regression: $(1-\tau_{t+1})\dfrac{e^E_{t+1}}{P_t} - \pi^E_{t+1} = \beta_1 \times \left[(1-\tau_{t+1})\text{10-year T-Yield} - \pi^E_{t+1}\right] + \beta_2 \times \pi^E_{t+1} + \upsilon_{i\,t+1}$

The variable $ey^E_{i\,t+1}$ is the after-tax real forward earnings yield adjusted for growth opportunities defined in Appendix E. r^E_{t+1} stands for the after-tax real expected stock yield. π^E_{t+1} is the one-year expected rate of inflation. $\dfrac{e^E_{t+1}}{P_t}$ is the nominal forward earnings yield on the S&P 500. τ_{t+1} is the blended tax rate on equity returns. Regression coefficients are obtained using Newey-West estimation, which corrects for possible autocorrelation up to 4 lags and for heteroskedasticity. We omit the intercept in the regressions because of the problem of collinearity with the real after-tax yields that are near constant. In parentheses are the t-statistics for these regressions. The F-statistic is reported.The adjusted R-squared is obtained from ordinary OLS, as a measure of overall regression fit. The Akaike (AIC) and Bayesian (BIC) information criteria are reported. All regressions assume that the required yield is 2.21%. The parameters γ_1 and γ_2 represent the PVGO speed of mean reversion respectively from above and below. A coefficient $\gamma = 1$ represents immediate mean reversion of PVGO to zero. RYT predicts that the coefficients for each regression should satisfy $\beta_1 = 1$ and $\beta_2 = 0$.

Periods	Variables	Fed Model	RYT without Treasury arbitrage		RYT with Treasury arbitrage	
			$\gamma_1 = 0.44$; $\gamma_2 = 0.68$	$\gamma = 1$	$\gamma_1 = 0.44$; $\gamma_2 = 0.68$	$\gamma = 1$
Q4 1953- Q3 2006 Obs. (212)	Slope β_1	0.80 (7.2)	1.11 (10.3)	0.99 (9.1)	1.10 (11)	0.99 (9.9)
	Slope β_2	0.26 (4.7)	-0.03 (-0.5)	-0.01 (-0.2)	-0.11 (-1.6)	-0.08 (-1.2)
	F-stat	143	194	164	254	217
	R-squared	74%	87%	84%	88%	86%
	AIC, BIC	-1243; -1236	-1351; -1345	-1355; -1348	-1379; -1372	-1382; -1375
Q4 1978- Q3 2006 Obs. (112)	Slope β_1	0.63 (5.8)	0.86 (7.8)	0.68 (6.5)	0.92 (10.4)	0.75 (8.6)
	Slope β_2	0.27 (4.4)	0.13 (1.5)	0.17 (2.0)	0.02 (0.3)	0.07 (1.1)
	F-stat	159	186	146	300	215
	R-squared	87%	90%	89%	94%	92%
	AIC, BIC	-738; -732	-750; -744	-756; -751	-805; -799	-803; -798

Figure 3: S&P 500 forward earnings yield: RYT formula vs. Fed model. Period Q4 1953- Q3 2006.

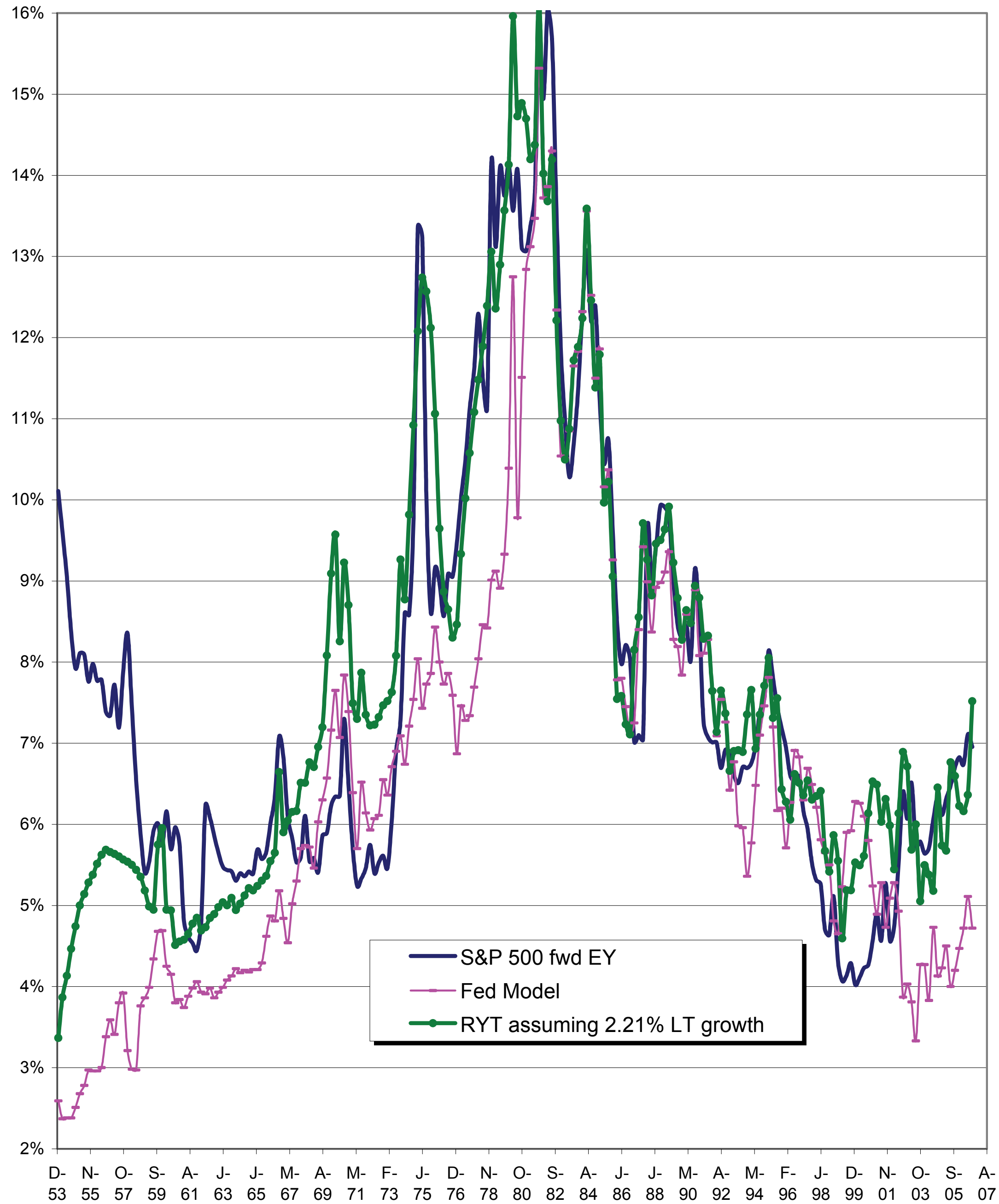


Figure 4: Measures of Real Growth of S&P 500 Book Value Per-Share. Period Q4 1953- Q3 2006.

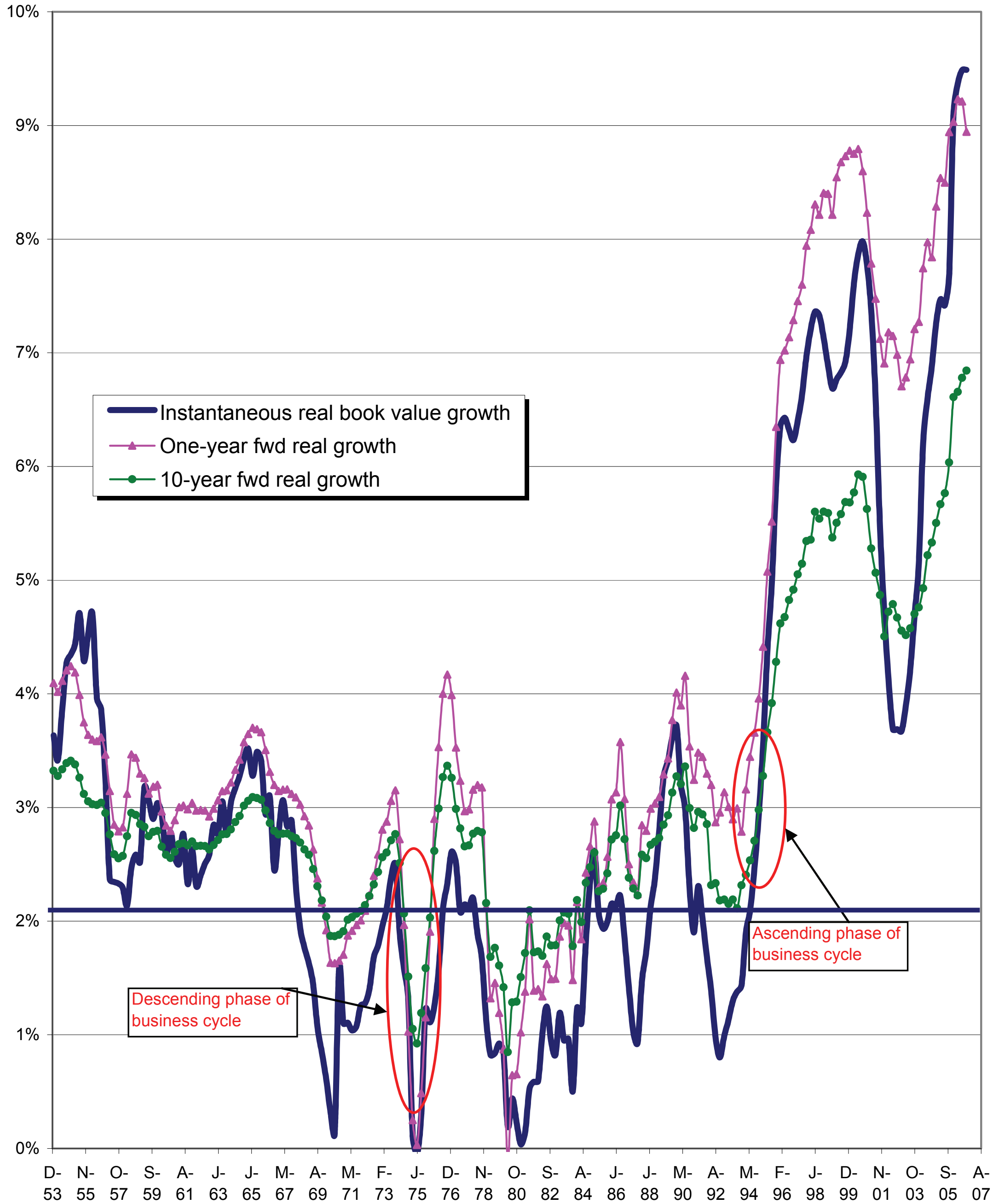


Table 4: Estimation of After-Tax Real Treasury Yields and Spread Using RYT

Regressions for Treasury yields: $r_{t+1}^{j}-\pi_{t+1}^{E}=\beta^{j}\left(g+I_{t+1}^{j}-B_{t+1}^{j}-(\lambda_{j}-1)\Phi_{t+1}\right)+\varepsilon_{t+1}^{j}$, for j = 1, 10

$r_{t+1}^{j}-\pi_{t+1}^{E}=\beta^{j}\left(g+I_{t+1}^{j}\right)+\varepsilon_{t+1}^{j}$, for j = 30

Regression for Yield Spread: $r_{t+1}^{10}-r_{t+1}^{1}=\beta\left(B_{t+1}^{1}-B_{t+1}^{10}+I_{t+1}^{10}-I_{t+1}^{1}-\Phi_{t+1}\right)+\upsilon_{t+1}$

The variable r_{t+1}^{j} stands for the nominal after-tax yield on a Treasury of maturity *j*. π_{t+1}^{E} stands for the one-year expected inflation. *g* represents the rate of long-term real GDP/capita growth. I_{t+1}^{j} represents the inflation risk premium for a Treasury of maturity *j*. Φ_{t+1} stands for the fear premium. λ_{j} represents the net effect of the fear premium on a Treasury of maturity *j*. Real variables are calculated by subtracting the one-year rate of expected inflation. Tax rates are average marginal (non-deferred) interest tax rates. Regression coefficients are obtained using Newey-West estimation, which corrects for possible autocorrelation up to 4 lags and for heteroskedasticity. We omit the intercept in the regressions because of the problem of collinearity with the real after-tax yields that are near constant. In parentheses are the t-statistics for these regressions. The F-statistic is reported. The adjusted R-squared is obtained from ordinary OLS, as a measure of overall regression fit. The Akaike (AIC) and Bayesian (BIC) information criteria are reported. All regressions assume that the required yield is 2.21%. The yield spread is the difference between the after-tax nominal 10-year Treasury yield minus the 1-year Treasury yield, because by assuming expected inflation is the one-year forecast for both maturities, the spread is equivalent to the difference between real after tax yields. The regression coefficients should satisfy βj=1 in all regressions for all *j*s.

Periods	Variables	1-Year Treasury	10-Year Treasury	30-Year Treasury	Yield Spread
Q4 1953- Q3 2006 Obs. (212)	Slope	0.76 (11.2)	0.93 (18.0)	N/A	0.90 (9.5)
	F-stat	126	325		91
	R-squared	66%	86%		58%
	AIC, BIC	-1368; -1364	-1454; -1450		-1548; -1549
Q4 1978- Q3 2006 Obs. (112)	Slope	0.98 (9.5)	1.04 (14.2)	0.91 (11.0)	0.92 (9.1)
	F-stat	90	203	122	82
	R-squared	66%	86%	80%	69%
	AIC, BIC	-724; -721	-757; -754	-750; -747	-805; -802

Figure 5: The Fear-Based Risk Premium. Q2 1992- Q3 2006

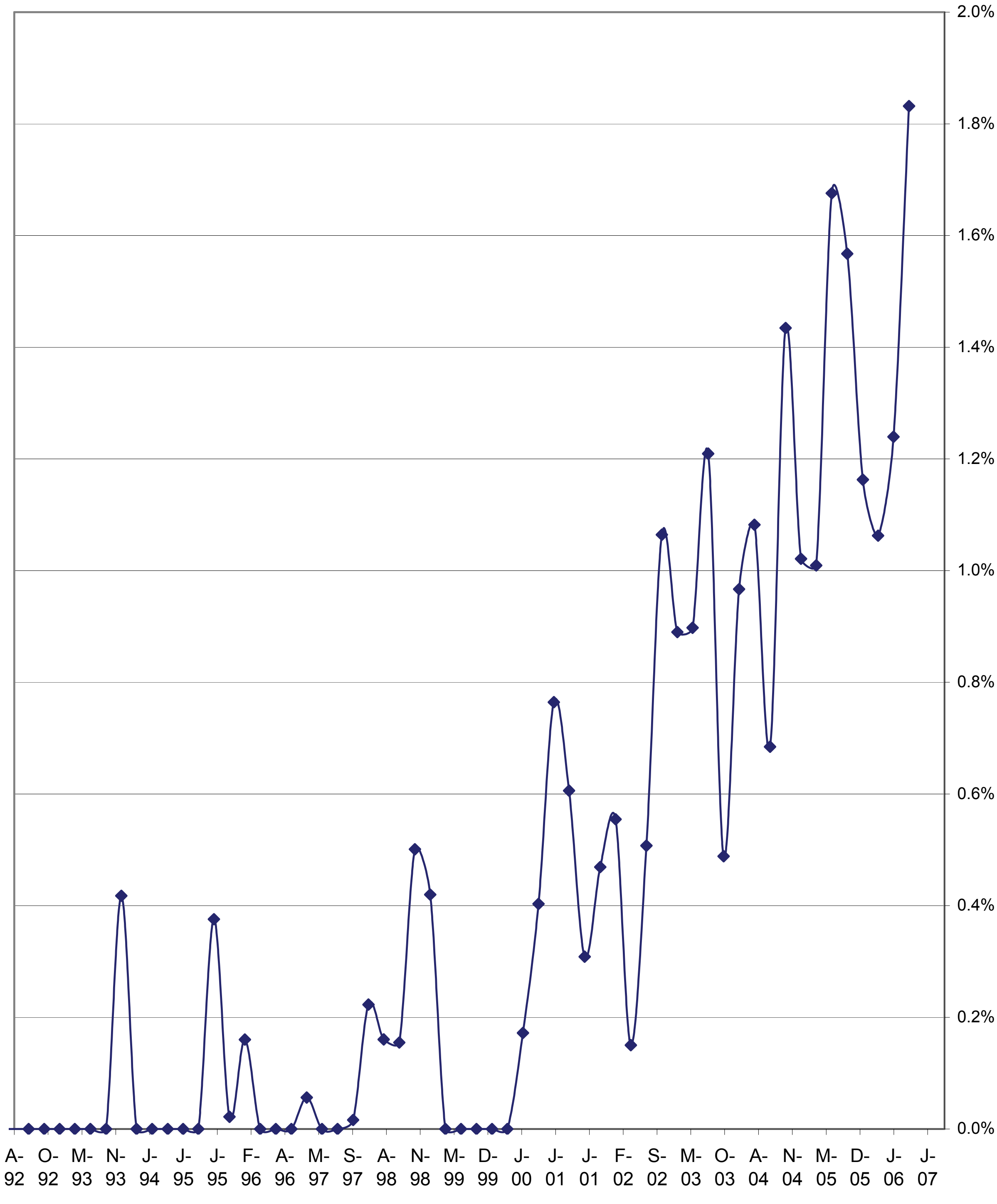


Figure 6a: RYT predicted after-tax nominal 1-year Treasury. Q4 1953- Q3 2006.

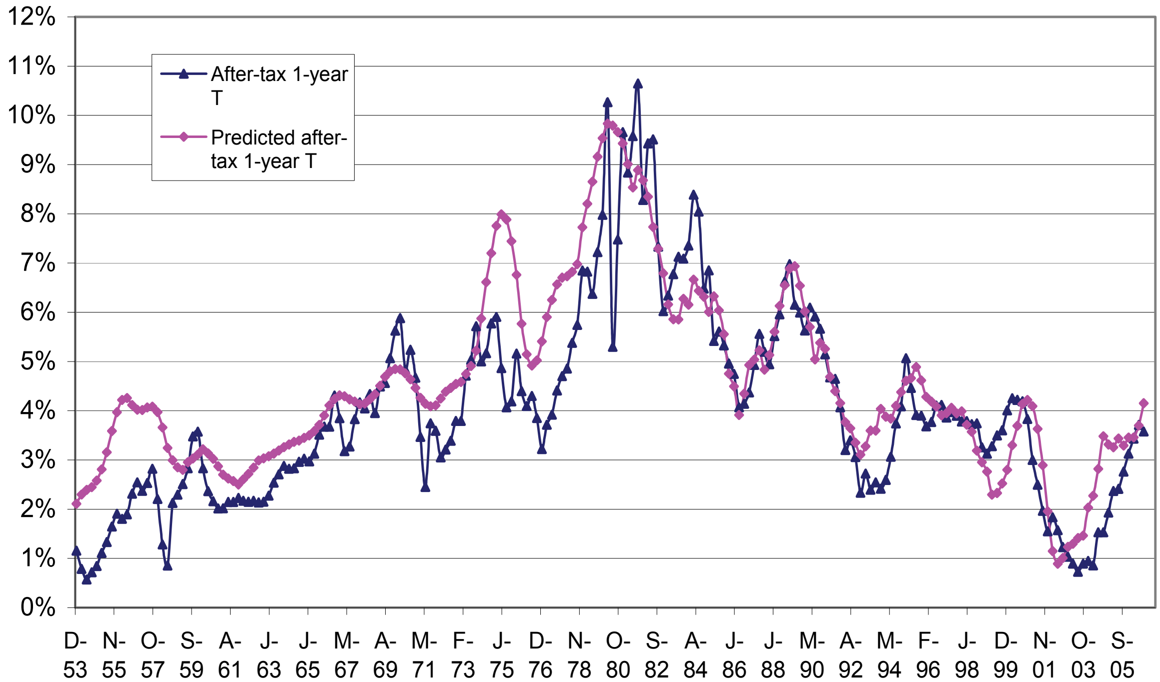


Figure 6b: RYT predicted after-tax nominal 10-year Treasury. Q4 1953- Q3 2006.

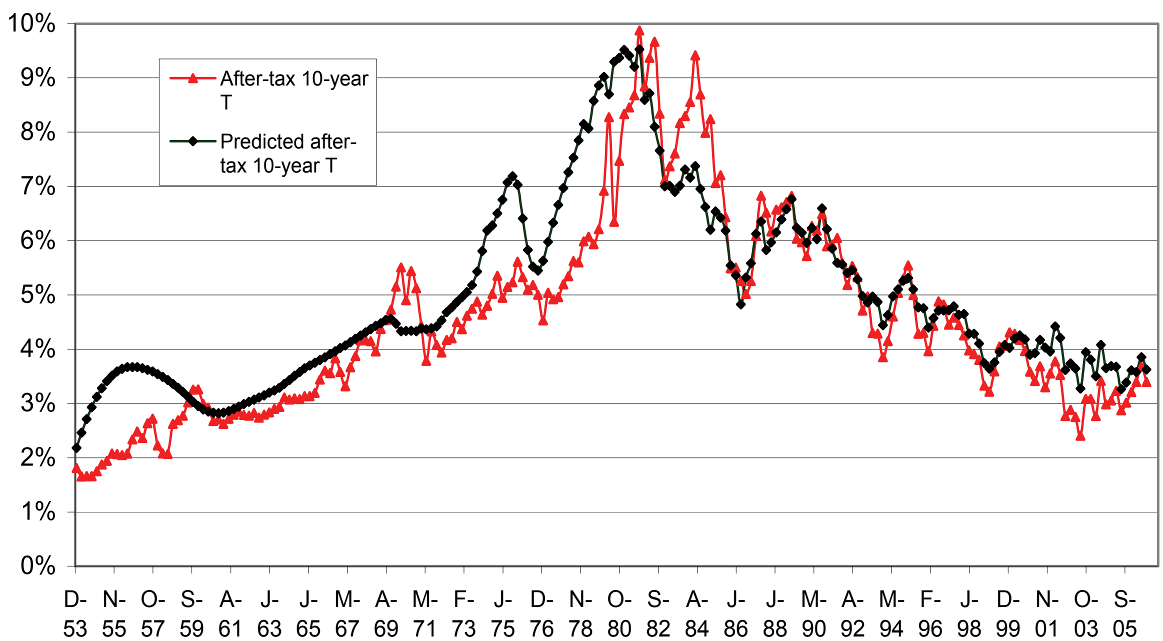


Figure 6c: RYT predicted after-tax nominal 30-year Treasury. Q1 1977- Q3 2006.

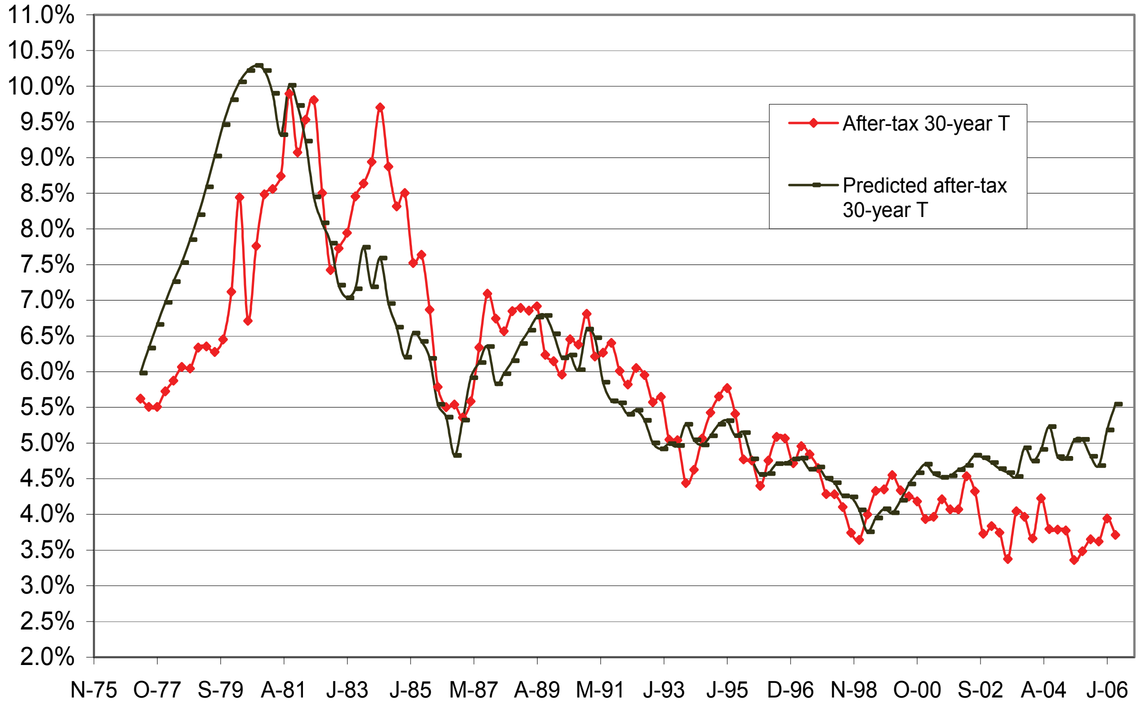


Figure 6d: RYT predicted after-tax 10-year minus 1-year Treasury spread. Q4 1953- Q3 2006.

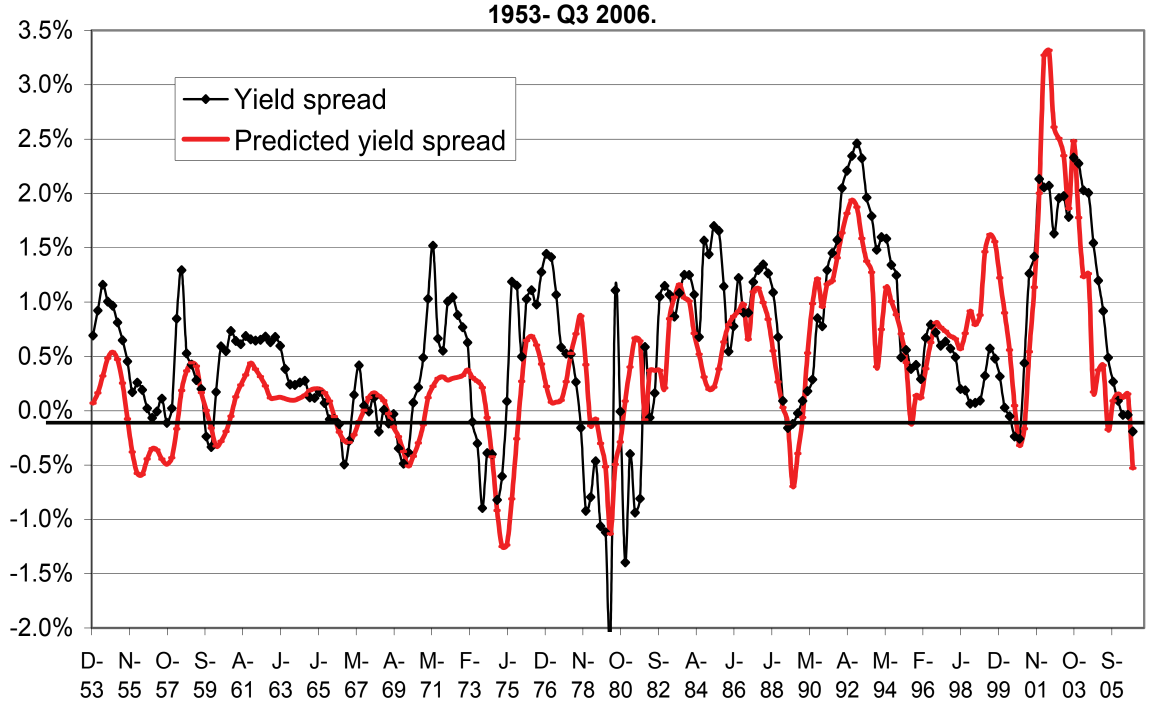
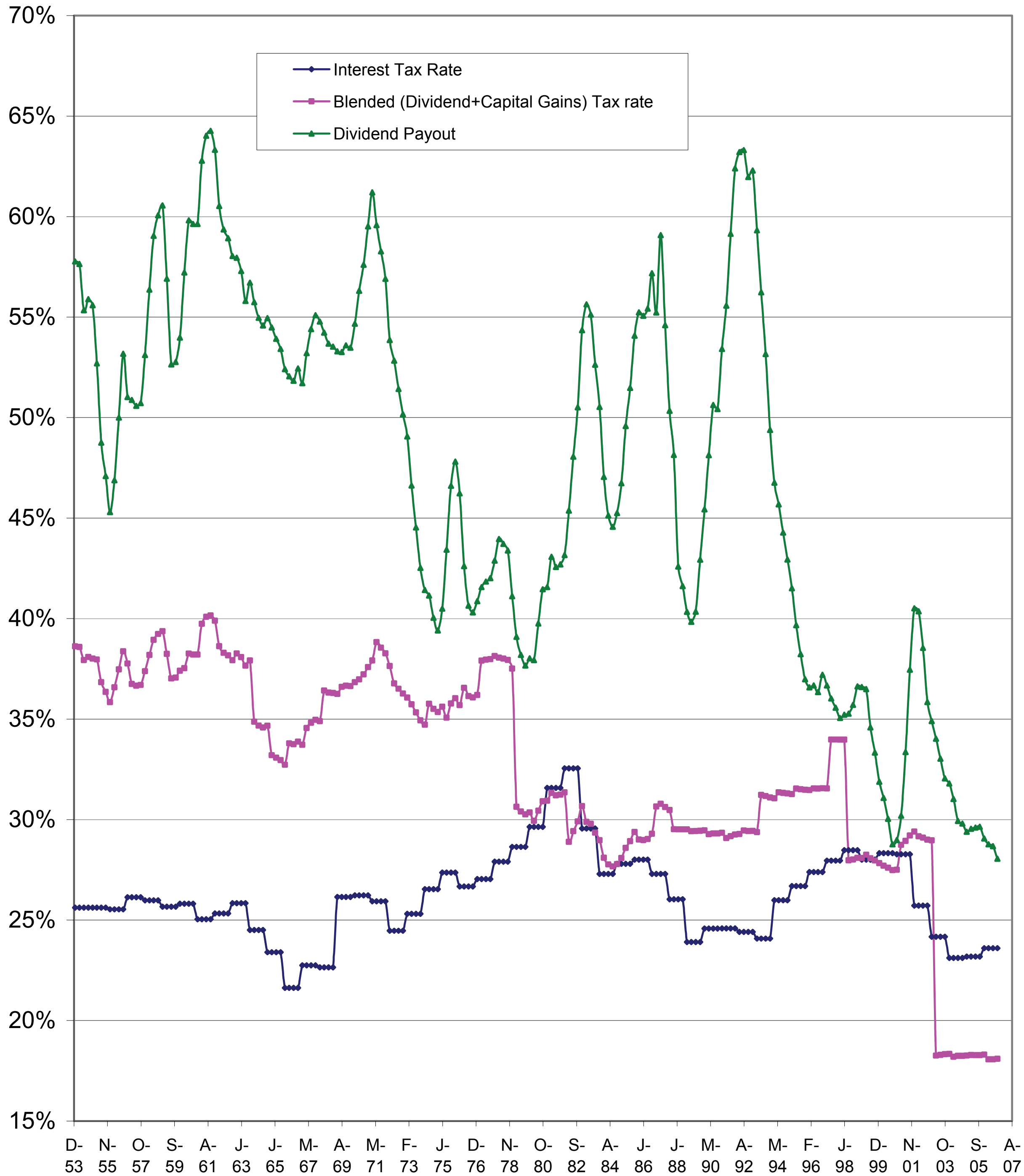


Figure 7: S&P 500 dividend payout, federal interest income tax rates and blended (dividend and LT capital gains) average marginal tax rates. Q4 1953- Q3 2006.



APPENDIX A: Description of Data and Variables

Treasury Yields: on the one-, ten- and thirty-year Treasuries are respectively series GS1, GS10 and GS30 and are obtained from the St. Louis Federal Reserve FRED II database website http://research.stlouisfed.org/fred2/categories/115. The series are reported on a monthly basis and used on a quarterly basis for our study. These series are constant maturity rates. The thirty-year Treasury data is only available for the period 1977-2001. It was interrupted in 2001 and then came back in 2006. We fill in the missing years/quarters between 2001 and 2006 by assuming the twenty-year Treasury plus 5 bps (obtained as the average difference between the two maturities prior to 2001) is a good proxy for the thirty-year bond at that time. Real after-tax yields are computed by multiplying the nominal yield by (1-interest tax rate) and subtracting expected *one-year* inflation expectations from the one-year Treasury and the one-year expected inflation from the ten-year and thirty-year Treasuries (prior to 1991), and the ten-year CPI inflation forecast after 1991 (from the Survey of Professional Forecasters).

Expected Inflation: From Q4 1953 to Q4 1981, expected inflation estimates are business selling price expectations from the quarterly business survey conducted by Fortune magazine for the period 1947-1983 and reported in Thies (1986). We use these expectations up to 1981, as these estimates are more conservative than GDP deflator estimates for the high inflation period covering 1970-1981 and because selling price expectations are in general more attuned to downward competitive price pressures. From Q3 1981-Q3 2006 the expectations are from the Survey of Professional Forecasters available on a quarterly basis since 1970 from the Federal Reserve Bank of Philadelphia website. The instantaneous inflation rate $\pi_{t+\Delta}$ is calculated as the average of the one-year forecast and the actual current CPI inflation rate. Ten-year forecasts are CPI inflation forecasts from the Survey of Professional Forecasters, available only after 1991 on a yearly basis.

Inflation risk premium estimates: are end of year estimates from Durham (2006) Table 1, from end of 2000 to July 2006. These are based on the decomposition of Ten-year nominal rates by calibrating term structure models separately for the nominal Treasury and TIPS yield curves. The estimates are applied equally on a quarterly basis, in this article. For each maturity the inflation premium is defined as follows: $I_{t+1}^{1} = 0$; I_{t+1}^{10} = (1-interest tax rate) x Durham's estimates; $I_{t+1}^{30} = 1.2 \times I_{t+1}^{10}$.

Personal Tax Rates: In this article, we use current marginal tax rates and not deferred rates. Dividend, capital gains and interest tax rates are all yearly *average* marginal rates. Dividend income tax rates are from Estrella and Fuhrer (1983) for the period 1954-1978 (which appear to include state taxes) and from the NBER TAXSIM model (including state and federal) for the period Q4 1978- Q3 2006 available at http://www.nber.org/taxsim. Long-term marginal capital gains tax rates for the period 1954-1965 are from Auerbach and Poterba (1988) and from the NBER TAXSIM model for the period 1966-1978 (only federal taxes). These rates are adjusted upward by a factor of 29% to account for the state portion of taxes (calculated using post 1979 data). The tax rate on government bonds is taken to be the *maximum* between the series of interest tax rate from Estrella and Fuhrer (adjusted downward by a factor of 18% that is calculated using post 1979 data, to bring the rate down to the federal portion) combined with the TAXSIM federal rates estimates for Q4 1978- Q3 2006, in comparison with estimates of government interest tax rates from Jorgenson and Yun (2001) available from 1953-2000. Via this procedure, Jorgenson and Yun's estimates are found to dominate the period 1979-2000. All annual tax rates are applied on a quarterly basis.

Earnings per share (Trailing), Dividends and Payout Ratios: S&P 500 historical trailing (as-reported) earnings per share and dividends per share are for the period 1953-2006 and obtained from Shiller's website www.econ.yale.edu/~shiller/data.htm. On the other hand, from Q4 1978 on, we use quarterly S&P operating earnings (i.e. street earnings) rather than as-reported earnings to compute dividend payout ratios. This is because as-reported earnings are too sensitive to one-time asset write-downs and do not reflect intended payouts.

Earnings-per-share (forward): estimates are from Thomson Financial for the period Q4 1978- Q3 2006. Prior to Q4 1978, we use current year's trailing earnings per share as an estimate of expected earnings, for our stock market valuation model(s).

Book value of equity per share and growth rates (productivity growth measures): is computed based on surplus accounting by cumulating retained earnings per share from 1871 using data from Shiller's website at www.econ.yale.edu/~shiller/data.htm. The past year growth of book value per share (past-year productivity growth measure) is then calculated as the ratio of current book value divided by the book value 4 quarters prior, minus 1. The *expected* growth in book value (one-year forecast of productivity growth) g_{t+1}^{E} is then computed as the ratio of expected retained earnings divided by current book value. Expected retained earnings are computed as the product of forward EPS times one minus a three-year moving average of past quarterly payout ratios. We use actual (ex-post) one-year ahead earnings per share to compute g_{t+1}^{E} prior to Q4 1978. After that, we use Thomson's forward earnings per share till Q3 2006.

S&P 500 prices and real after–tax total return: are end-of month ^GSPC price series obtained from Yahoo finance and selected on a quarterly basis for the period Q4 1953-Q3 2006. The total return is computed by annualizing quarterly capital gains plus dividend yields. The tax rates applied are the long-term capital gains and dividend income tax rates. Because the total return is ex-post we use actual inflation measure (GDP deflator) to get the real after-tax return.

S&P 500 present value of growth opportunities (actual): are calculated using the formula $PVGO_{t+1} = \frac{\left[P_{t+1} - P_t - (1 - b_{t+1})e_{t+1}\right]}{k_{t+1}^{E}}$ (derived from Equation 2 in the text) where P_{t+1} is the actual S&P 500 price and e_{t+1} is the actual historical earnings per share. The required return is assumed to be the same as our required yield that incorporates Treasuries arbitrage and all the various risk premia, but does not incorporate the abnormal growth of earnings component to avoid circularity.

APPENDIX B: Expected PVGO Derivation

Let us begin with the pricing equation (3):

$$P_t = (1-\tau_{t+1})\frac{e_{t+1}^E}{k_{t+1}^E} + (1-\tau_{c\ t+1})PVGO_{t+1}^E \tag{B1}$$

Updating equation (B1), taking expectations on both sides and using the law of iterated expectations, equation (B1) becomes:

$$P_{t+1}^E = \left[(1-\tau_{t+2})E_t\left(\frac{e_{t+2}}{k_{t+2}}\right) + (1-\tau_{c\ t+2})E_t\left(PVGO_{t+2}\right)\right] \tag{B2}$$

Moreover, the dynamics of PVGO expectations is given by equation (4):

$$PVGO_{t+1}^E = (1-\gamma_i)PVGO_t \tag{B3}$$

Updating equation (5) and again using the law of iterated expectations, equation (B3) becomes:

$$E_t\left(PVGO_{t+2}\right) = (1-\gamma_i)PVGO_{t+1}^E \tag{B4}$$

Subtracting (B1) from (B2), using (B4), and assuming that $0 \le \gamma_i < 1$, and that the capital gains and blended tax rates are expected to remain constant[51] beyond period *t+1*, we get:

$$P_{t+1}^E - P_t = (1-\tau_{t+1})\frac{e_{t+1}^E}{k_{t+1}^E}\left[E_t\left(\frac{e_{t+2}}{k_{t+2}}\right)\frac{k_{t+1}^E}{e_{t+1}^E} - 1\right] - (1-\tau_{c\ t+1})\gamma_i PVGO_{t+1}^E \tag{B5}$$

By definition, $PVGO_{t+1}^E = \frac{\left[P_{t+1}^E - P_t - (1-b_{t+1})e_{t+1}^E\right]}{k_{t+1}^E}$, substituting this expression in (B5), and after several manipulations, we get:

$$PVGO_{t+1}^E = (1-\tau_{t+1})\frac{e_{t+1}^E}{k_{t+1}^E} \times \frac{\left[E_t\left(\frac{e_{t+2}}{k_{t+2}}\right)\frac{k_{t+1}^E}{e_{t+1}^E} - 1 - \frac{(1-b_{t+1})k_{t+1}^E}{(1-\tau_{t+1})}\right]}{k_{t+1}^E + \gamma_i(1-\tau_{\mathrm{c}\ t+1})} \tag{B6}$$

[51] Interestingly, this does *not* imply that the payout ratio must remain constant. However, for all intents and purposes if investors do not anticipate changes in the tax code, we are effectively assuming that the payout ratio will stay constant beyond period t+1, conditional on the information available at time t.

To simplify formula (B6) further, let us define the expected book value B_{t+1}^{E} (conditional on information at time *t*) of the index using surplus accounting by $B_{t+1}^{E} = B_t + (1-b_{t+1})e_{t+1}^{E}$, where B_t stands for the actual book value at time t. Using our previous definition of the expected sustainable growth rate (g_{t+1}^{E}) we see that the growth in book value, $\frac{B_{t+1}^{E}}{B_t} - 1 = \frac{(1-b_{t+1})e_{t+1}^{E}}{B_t} = g_{t+1}^{E}$. Furthermore, one plus the expected growth rate of earnings (conditional on information available at time t) is then given by $\frac{E_t(e_{t+2})}{e_{t+1}^{E}} = \frac{E_t(g_{t+2})}{g_{t+1}^{E}} \times \frac{(1-b_{t+1})}{(1-b_{t+2})} \times (1+g_{t+1}^{E})$. As previously said, we assume that $b_{t+1} = b_{t+2}$, i.e. conditional on information available at time t, investors believe that the payout ratio will not change in periods t+1 and t+2.

Also, let us approximate $E_t\left(\frac{e_{t+2}}{k_{t+2}}\right)\frac{k_{t+1}^{E}}{e_{t+1}^{E}} - 1 \approx \frac{E_t(e_{t+2})}{E_t(k_{t+2})}\frac{k_{t+1}^{E}}{e_{t+1}^{E}} - 1$ (which is we realize is not exact due to Jensen's inequality). Furthermore, we also posit that $\frac{E_t(g_{t+2})}{g_{t+1}^{E}} \approx \frac{E_t(k_{t+2})}{k_{t+1}^{E}}$, which in the context of the real after-tax required return equaling long-term GDP/capita growth, essentially necessitates that changes in the sustainable growth rate are mostly driven by (small) changes in inflation, and not by changes in real productivity.

These two last assumptions together lead us to the result that $E_t\left(\frac{e_{t+2}}{k_{t+2}}\right)\frac{k_{t+1}^{E}}{e_{t+1}^{E}} - 1 \approx g_{t+1}^{E}$. Using, this last approximation and factoring out the retention ratio in equation (B6), leads to our final expression:

$$PVGO_{t+1}^{E} = (1-\tau_{t+1})\frac{e_{t+1}^{E}}{k_{t+1}^{E}} \times \frac{(1-b_{t+1})\left[\frac{g_{t+1}^{E}}{(1-b_{t+1})} - \frac{k_{t+1}^{E}}{(1-\tau_{t+1})}\right]}{k_{t+1}^{E} + (1-\tau_{c\ t+1})\gamma_i} \qquad \text{(B7)}$$

Finally, it is important to note that to be consistent with condition spelled out in condition (4) we must guarantee that $PVGO_{t+1}^{E}$ is positive or negative. As long as b_{t+1}>

$\tau_{t+1} \left[\frac{g_{t+1}^{E}}{(1-b_{t+1})} - \frac{k_{t+1}^{E}}{(1-\tau_{t+1})} \right]$>0 is implied by g_{t+1}^{E} greater or close to k_{t+1}^{E} and changes in the sustainable growth rate come from changes in inflation and not real productivity. Otherwise negative PVGOs will require that g_{t+1}^{E} be much smaller than k_{t+1}^{E}.

APPENDIX C: The Growth of Equity Shares Must Equal Population Growth

Over the period 1929-2006, S&P500 earnings per-share (EPS) grew at a rate of 5.23% while GDP grew at 6.50%. Since the ratio of aggregate corporate profits to GDP must be constant in the long-run, net new share growth is obtained as the difference between GDP growth and earnings per-share growth. Over the period, net share growth was 1.27% or about equal to the 1.38% population growth. Similarly, over the period 1946-2006, the growth in total value of corporate equity was 8.98% (using the Federal Flow of Funds), whereas it was 7.67% for the S&P 500 over the same period.

Because the S&P500 has been a relatively constant fraction of the overall market value (about 60%), and the index is on a per-share basis, it is evident that the difference of 1.31% represents net share growth, again nearly equal to population growth.

It is also theoretically logical that net new share growth should equal population growth. In order for new shares to be purchased by individual investors (net of asset substitution), the price per-share cannot grow faster than wage income per-capita in the long run. Otherwise, *new* shares would eventually become unaffordable. Since total wages and total market value both grow at the rate of GDP, this entails that share growth must at least be population growth.

On the other hand, share growth permanently in excess of population growth would shrink earnings per-share. The return per share would go down, since EPS growth would be slower and free-cash flows per-share payable as dividends would also be smaller. This means that investors would bid down stock prices. Furthermore, stock dilution by bringing down the ex-post total return may be viewed as increasing the downside risk for investors. In that case, the equity premium may *increase*, which leads to further price drops and thus to a non-equilibrium outcome.

APPENDIX D: The Interaction between Tax Policy, Growth and Dividends

In accordance with the Maturity Hypothesis of dividend policy (Grullon, Michaely and Swaminathan; 2002), we postulated in the text that S&P 500 index companies generate free cash flows that cannot be reinvested in strictly positive net present value projects. Stated differently, investors are better-off receiving these free-cash flows as dividends (or share repurchases) because reinvesting these cash flows in the S&P 500 would not produce a greater sustainable earnings growth per share.

In connection with this phenomenon, we introduce here a new assumption that at the macroeconomic level, *tax policy on equity returns must be designed so as not to hamper the sustainable rate of corporate earnings growth per share*.[52] This hypothesis may appear radical, and it should be formally investigated via an analysis of a political equilibrium that determines tax rates in the nation, which is beyond the scope of this study. However, we assert here that individuals and corporations are able to lobby the government to still achieve the maximum economically feasible growth of corporate earnings per share.

This in fact is possible as long as free cash flows are available that cannot produce faster growth for the corporate sector, and therefore are sufficient to cover the personal tax bill generated by investing in a share of the index, for the highest marginal tax brackets. While we are not aware of studies directly supporting our view, several articles (Stokey and Rebelo, 1995; Slemrod and Bakija, 1996) have shown that contrary to common held beliefs the evidence is not clear that higher income tax rates reduce economic growth.

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[52] Obviously there have been historical periods when taxes were predatory. For example, top income marginal rates were sharply raised between 60 and 80% before each world war. We consider that in time of political stability our hypothesis is reasonable. Implicitly this assumption also entails that *corporate* taxes are not impeding growth as well, as free cash flows must be available before *corporate* taxes. While it may seem that this hypothesis is not true for growth companies, but in fact, as long as earnings are reinvested and gains are unrealized, personal taxes do not hamper the growth rate of these companies.

The previous argument implies that the blended average marginal tax rate should always be smaller than the index's optimal payout ratio.[53] This is empirically confirmed over our sample period (see Figure 7). Formally, we assume $\tau_{t+1} \leq b_{t+1}^{*}$ where b_{t+1}^{*} is the optimal dividend payout related to the optimal policy defined earlier, with the maximum sustainable growth rate of earnings per share denoted as $g_{\max t+1}^{E} = (1 - b_{t+1}) \times ROE_{t+1}$, whenever $b_{t+1} \leq b_{t+1}^{*}$. Firstly, our hypothesis implies that in the long-run, mature firms choose the payout/retention ratio so as to maximize sustainable earnings per share growth. Changes in the relative dividend income vs. capital gains tax rates will only affect the portion of free cash flows that are distributed within the payout interval that allows for maximum growth.

In the long-run, as *t* goes to infinity, the maximum sustainable growth rate becomes $g_{\max t+1}^{E} = g + \overline{\pi}_{t+1}^{E}$, where g represents the long-run real GDP/capita growth rate and $\overline{\pi}_{t+1}^{E}$ represents the expected long-run inflation rate. Thus, in the long-run, the S&P 500 return on equity (ROE) drops if the retention ratio is larger than the optimal threshold. This is because the growth of earnings per share is capped by GDP/capita growth as shown in Section 3.1.[54]

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[53] This is assuming that earnings per share are a good proxy for (dollar dividends) + (dollar capital gains), which must be true in the long-run. As an aside, the government has an incentive to capture the entire non-productive free cash-flows as taxes. In that case, the effective payout ratio (computed as Dividend Yield/Total Return) should match the blended tax rate, which is confirmed historically over Q4 1953- Q3 2006 , since both variable averaged 32%.

[54] Arnott and Bernstein (2003) argue that higher dividend payouts are correlated with *higher* future earnings growth, which appears to contradict our optimal dividend policy. A potential explanation of their finding is that the dividend income tax rate *relative* to the capital gains tax rate has generally decreased, at least over Q4 1953- Q3 2006 . This means that consistent with our growth hypothesis, companies could *increase* their payout ratio without sacrificing their *maximum* sustainable growth, which can also vary because of growth opportunities. In other words, when the ROE goes up, the threshold payout ratio can also go *up*. It is interesting to note that during the period 1978-1981, according to Arnott and Bernstein's results, the payout ratio was moving *inversely* with future earnings growth. This corresponds to a period where the *relative* tax rate rose back up for dividends and both tax rates (dividend income and capital gains) went up too, which had the effect of contracting growth. It is likely that higher personal taxes would raise the hurdle rate for capital budgeting projects, thus free cash flows should shrink with higher taxes. Hence, in the *absence* of personal taxes, and with firms' PVGOs converging to zero, free cash flows would be driven down to zero.

APPENDIX E: Regressions Using After-Tax Real Yields

To avoid the problem of spurious regressions it is usually recommended to use stationary variables to test economic relationships. While we found that our nominal before-tax yield series are stationary over the sample period 1953-2006 this result does not hold over the subsample period Q4 1978- Q3 2006. On the other hand, our unit roots tests show that the after-tax real yield on Treasuries and the S&P 500 earnings yield and total market return are indeed stationary time-series with drifts in these subsamples at least at the 5% level.[55] Therefore, we use these variables to test our Required Yield Theory.

1. <u>Regressions Testing the Fit for the S&P 500 Earnings Yield</u>

Let us start from equation (8) in the main text, we are testing:

$$\frac{e_{t+1}^E}{P_t} = \beta \times \frac{Max\{R_{t+1}^E, r_{t+1}^1, r_{t+1}^{10}\}}{(1-\tau_{t+1})\left[1+(1-\tau_{c\ t+1})AEG_{i,t+1}^E\right]} + \varepsilon_t \text{ for } i = 1,2 \qquad \text{(D1)}$$

With β=1. Let $r_{t+1}^E = Max\{R_{t+1}^E, r_{t+1}^1, r_{t+1}^{10}\} - \pi_{t+1}^E$ denote the after-tax real expected yield that applies to stock valuation. Let $ey_{i\ t+1}^E = (1-\tau_{t+1})\left[1+(1-\tau_{c\ t+1})AEG_{i,t+1}^E\right]\frac{e_{t+1}^E}{P_t} - \pi_{t+1}^E$ denote the after-tax real forward earnings yield taking into account growth opportunities. We can rewrite equation (D1) above as:

$$ey_{i\ t+1}^E = \beta \times r_{t+1}^E - (1-\beta)\pi_{t+1}^E + v_{i\ t+1} \qquad \text{(D2)}$$

Where $v_{i\ t+1}^E = (1-\tau_{t+1})\left[1+(1-\tau_{c\ t+1})AEG_{i,t+1}^E\right]\varepsilon_{t+1}$. As long as $ey_{i\ t+1}^E$ and r_{t+1}^E are stationary variables, we can proceed with OLS to test our theory.[56] Thus, we implement the general test that:

$$ey_{i\ t+1}^E = \beta_1 \times r_{t+1}^E + \beta_2 \times \pi_{t+1}^E + v_{i\ t+1} \quad i=1,2 \qquad \text{(D3)}$$

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[55] We use a uniform transformation and express all real yields based on *one-year* inflation expectations. Our results regarding the rejection of non-stationarity of these transformed series are available upon request from the authors.

[56] It is not necessary to have expected inflation π_{t+1}^E being stationary for our results to hold.

With β_1= 1 and $\beta_2 = -(1-\beta_1)$, and where the index i applies to the mean reversion parameter from above or below. The reason why we omit the intercept in regression (D3) is to avoid a multicollinearity issue because we expect the real after-tax return r_{t+1}^E to be nearly constant. We are testing four different versions of (D3). Version 1 is assuming that $r_{t+1}^E = g$, i.e. that there is no Treasury arbitrage, but that PVGO mean reversion occurs with two speeds adopts two speeds γ_1= 43.6%, and γ_2= 68% respectively from above and below. Version 2 assume that in addition the reversion of PVGO is instantaneous, i.e. that γ=1 or that $AEG_{i,t+1}^E$= 0. Version 3 incorporates Treasury arbitrage and uses the two speeds γ_1= 43.6%, and γ_2= 68%. Version 4 still assumes Treasury arbitrage but instantaneous reversion of PVGO to zero.

Our treatment of the Fed model is to transform the variables on an after-tax and real basis, and compare its performance with our model in (D3). After transformation, the variables are found to be stationary with drift (except for expected inflation over Q4 1978- Q3 2006). The regression for testing the Fed model takes the form:

$$(1-\tau_{t+1})\frac{e_{t+1}^E}{P_t} - \pi_{t+1}^E = \beta_1 \times \left[(1-\tau_{t+1})\text{10-yearYield} - \pi_{t+1}^E\right] + \beta_2 \times \pi_{t+1}^E + \upsilon_{i\ t+1} \qquad \text{(D4)}$$

With β_1= 1 and $\beta_2 = -(1-\beta_1)$. Where the tax rate applied is the blended rate on equity returns. Again, the reason why we omit the constant term in the regression is because the real after tax yield on the 10-year Treasury is fairly constant, and including a constant term would generate a multicollinearity problem.

2. Regressions Testing Treasury Yields and Yield Spread

To test our theory of yield determination we start from the equations in Section 6 describing the behavior of the after-tax nominal yield on Treasuries for horizons of 1-year, 10-year and 30-year. Our treatment of real yields differs from the text as we are exclusively using one-year ahead expected inflation to transform these yields on real yields. The reason is that we are applying a uniform transformation of these variables to render them stationary; whereas in the text, we correct for the expected inflation over the horizon that matches the Treasury maturity. As in the text, we assume here that the fear premium is fully symmetrical for the 10-year Treasury yield and the stock market return (λ_{10}= 2). Hence, we test:

$$r_{t+1}^{j} - \pi_{t+1}^{E} = \beta^{j}\left(g + I_{t+1}^{j} - B_{t+1}^{j} - (\lambda_{j} - 1)\Phi_{t+1}\right) + \varepsilon_{t+1}^{j}, \text{ for } j = 1, 10 \qquad \text{(D5)}$$

To avoid circularity regarding the joint determination of the fear premium and the yield on the 30-year Treasury, the regression equation is:

$$r_{t+1}^{j} - \pi_{t+1}^{E} = \beta^{j}\left(g + I_{t+1}^{j}\right) + \varepsilon_{t+1}^{j}, \text{ for } j = 30 \qquad \text{(D6)}$$

where the slope coefficients should satisfy $\beta^{j}=1$ for j=1, 10, 30. The error terms are assumed white noises and the error term ε_{t+1}^{30} is correlated with the fear premium (when negative). We can test these relationships as long as the real after-tax yields and the composite exogenous variables are stationary, which is confirmed empirically in all our subsamples. The regression for the yield spread is:

$$r_{t+1}^{10} - r_{t+1}^{1} = \beta\left(B_{t+1}^{1} - B_{t+1}^{10} + I_{t+1}^{10} - I_{t+1}^{1} - \Phi_{t+1}\right) + \upsilon_{t+1} \qquad \text{(D7)}$$

where $B_{t+1}^{1} = Max\left\{g_{t+1}^{E} - g_{t+\Delta}, g_{t+1}^{10} - g_{t+\Delta}\right\}$ and $B_{t+1}^{10} = Max\left\{g + \pi_{t+1}^{10} - g_{t+1}^{10}, 0\right\}$ respectively are the business cycle risk premia corresponding to each maturity. Equation (D7) is different from equation (10) in the text, which assumes different expected inflation rates for each maturity. Because we apply a single measure of inflation expectations (one-year ahead) for both maturities, this is equivalent to testing the yield spread as a function of after-tax *nominal* yields in equation (D7). Here again our theory predicts that the coefficient β=1.